

EXAM FOR STOCHASTIC MODELS IN DISCRETE TIME

3.75 ECTS

Master's program of Financial Mathematics
January 8, 2011, 9.00 – 13.00

Max number of points: 30.

Halmstad University grading bounds: 12p \Rightarrow grade 3, 18p \Rightarrow grade 4, 24p \Rightarrow grade 5.

ECTS bounds: 12p \Rightarrow grade E, 15p \Rightarrow grade D, 18p \Rightarrow grade C, 21p \Rightarrow grade B, 24p \Rightarrow grade A.

Allowed aids: Summary of formulae attached to the exam, calculator and dictionary.

Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet.

The proper solutions will be available on the internet at <http://www.hh.se/staff/erja> \rightarrow
 \rightarrow Teaching \rightarrow Financial Mathematics \rightarrow Stochastic models \rightarrow Previous exams

1. Assume that $X = \{(X_n, \mathcal{F}_n) : n \geq 0\}$ is a stochastic sequence with $E(|X_n|) < \infty$. Prove that if X is a local martingale, then X is a generalized martingale. (4p)
2. Let $\{X_t : t \in \mathbb{Z}\}$ be an $ARMA(1, 2)$ process defined by $X_t + 0.5X_{t-1} = 0.5\epsilon_{t-2} + b_0\epsilon_t$ for all t where $\{\epsilon_t\}$ is white noise and $D(X_t) = 0.375$. Calculate the
 - (a) positive value of the parameter b_0 . (4p)
 - (b) probability $P(2X_t + X_{t-1} \leq 1)$. (4p)
3. Consider the $ARCH(1)$ process $\{h_t : t \in \mathbb{Z}\}$ with $a_0 = a_1 = \frac{1}{2}$, $E(h_t^2) = 1$ and $E(h_t^4) = 9$. Calculate
 - (a) $C(h_t^2, h_{t+1}^2)$. (4p)
 - (b) the conditional density function $f_{h_t|h_{t-1}}(h|\tilde{h})$. (4p)
4. Let $\{N_t : t \in \mathbb{R}^+\}$ be a Poisson process with intensity $\lambda = 1$. Prove that the process $\{Y_t : t \in \mathbb{R}^+\}$ defined by $Y_t = N_{t+1} - N_t$ for all $t \in \mathbb{R}^+$ is weakly stationary. (5p)
5. Consider the random process $\{\theta_n : n \in \mathbb{Z}\}$ which is equipped with the flow $\{\mathcal{F}_n : n \in \mathbb{Z}\}$ where $\mathcal{F}_n = \sigma(\theta_n, \theta_{n-1}, \theta_{n-2}, \dots)$. Assume that $E(\theta_{n+1}^k | \mathcal{F}_n) = \theta_{n-1}^k$ for all $k = 1, 2, 3, \dots$, that $E(|\theta_n|) < \infty$, $E(\theta_n^2) < \infty$ and $|C(\theta_n, \theta_{n+1})| < \infty$ for all $n \in \mathbb{Z}$. Then, is the process $\{(\theta_{n+1} + \theta_n)^2 : n \in \mathbb{Z}\}$ a martingale, a supermartingale, a submartingale or neither of these? (5p)

GOOD LUCK!