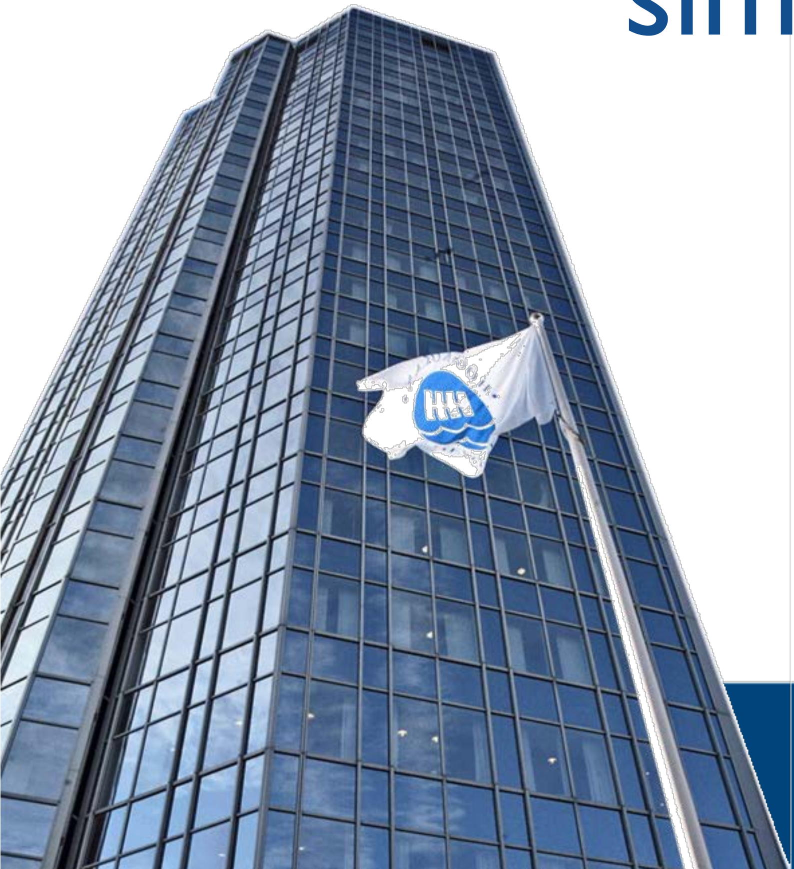
Acumen: a domain specific language for simulating hybrid systems



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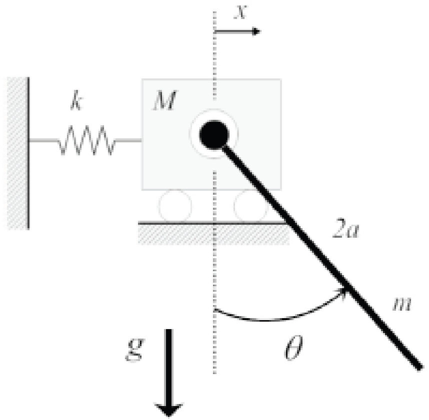
- April 2012: PhD in signal and systems
  - Wearable inertial sensors for gait analysis

- June 2012: joined Effective Modeling Group
  - Collaboration PSU
  - Virtual test-bed for smart-grids in Acumen

Overview

- Background
- Examples
- Future work

The model to simulation gap



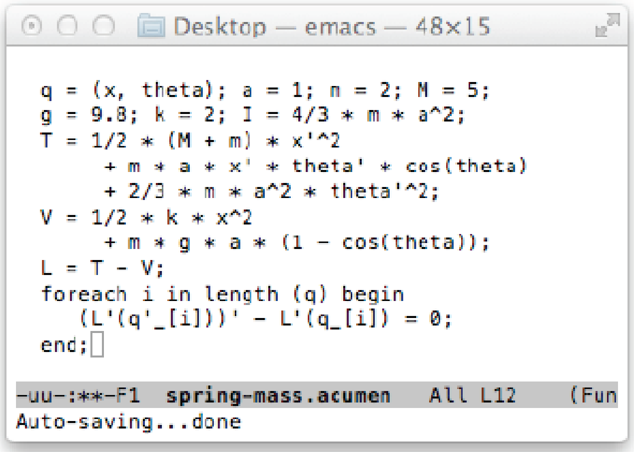
$$q = [x, \theta] \quad a = 1 \quad m = 2 \quad M = 5$$

$$g = \frac{49}{5} \quad k = 2 \quad I = \frac{4}{3}ma^2$$

$$T = \frac{1}{2}(M+m)\dot{x}^2 + ma\dot{x}\dot{\theta}\cos(\theta) + \frac{2}{3}ma^2\dot{\theta}^2$$

$$V = \frac{1}{2}kx^2 + mga(1 - \cos(\theta)) \quad L = T - V$$

$$\forall i \in \dim(q) \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$



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Desktop — emacs — 48×15

q = (x, theta); a = 1; m = 2; M = 5;
g = 9.8; k = 2; I = 4/3 * m * a^2;
T = 1/2 * (M + m) * x'^2
     + m * a * x' * theta' * cos(theta)
     + 2/3 * m * a^2 * theta'^2;
V = 1/2 * k * x^2
     + m * g * a * (1 - cos(theta));
L = T - V;
foreach i in length (q) begin
   (L'(q'_[i]))' - L'(q_[i]) = 0;
end;

-uu-:**-F1  spring-mass.acumen    All L12    (Fun
Auto-saving...done
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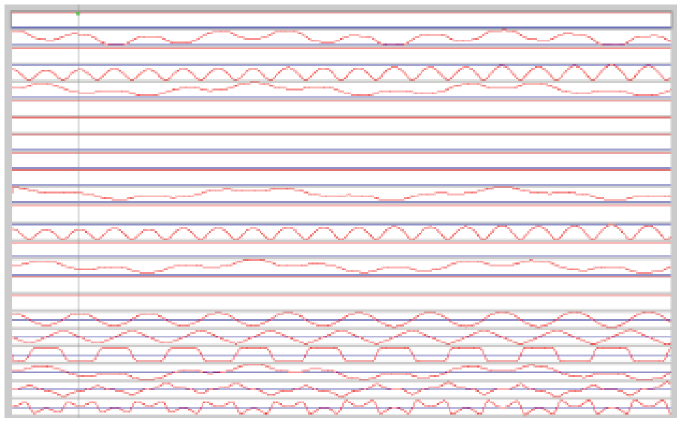
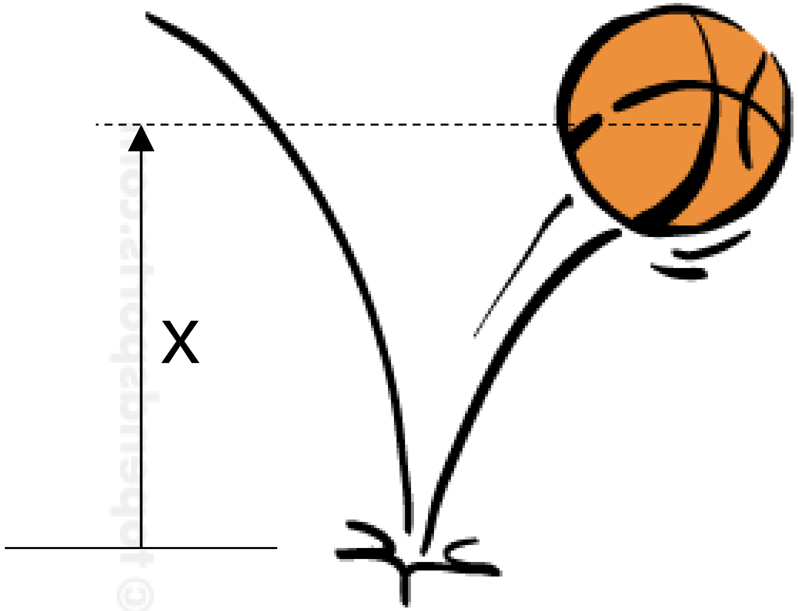


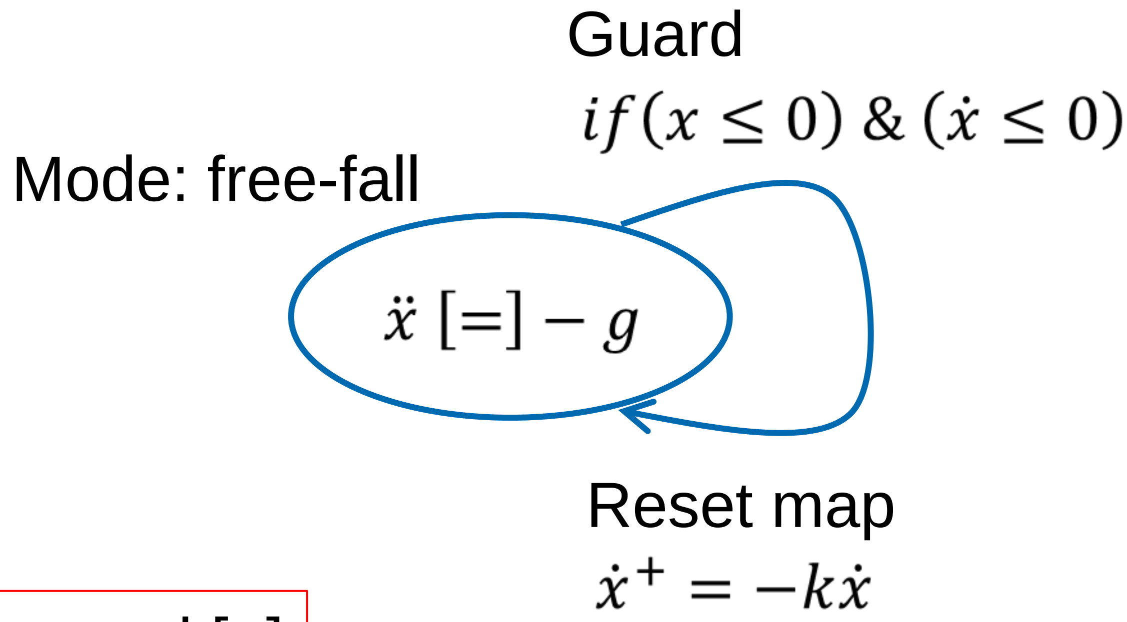
Acumen

- DSL for modeling and simulating hybrid systems
  - Continuous and discrete behavior
  - Physical system and digital components
- Close the gap between modeling and simulation
  - Mathematical equations as executable models

Bouncing ball

- Typical hybrid system example

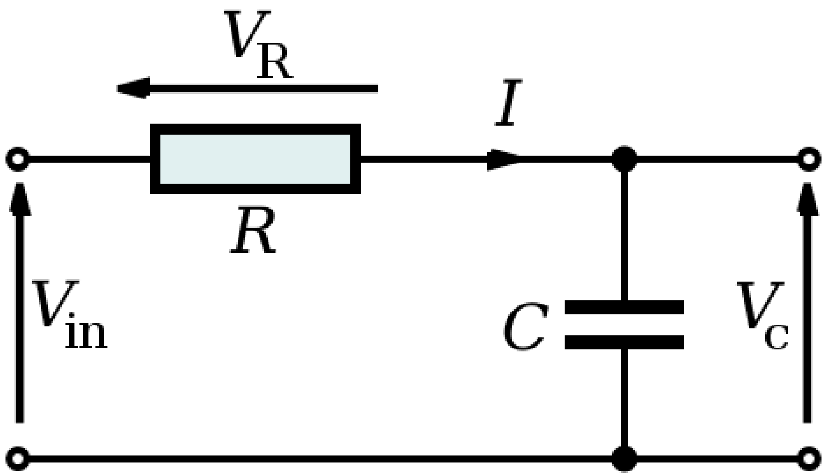




Note: = and [=]

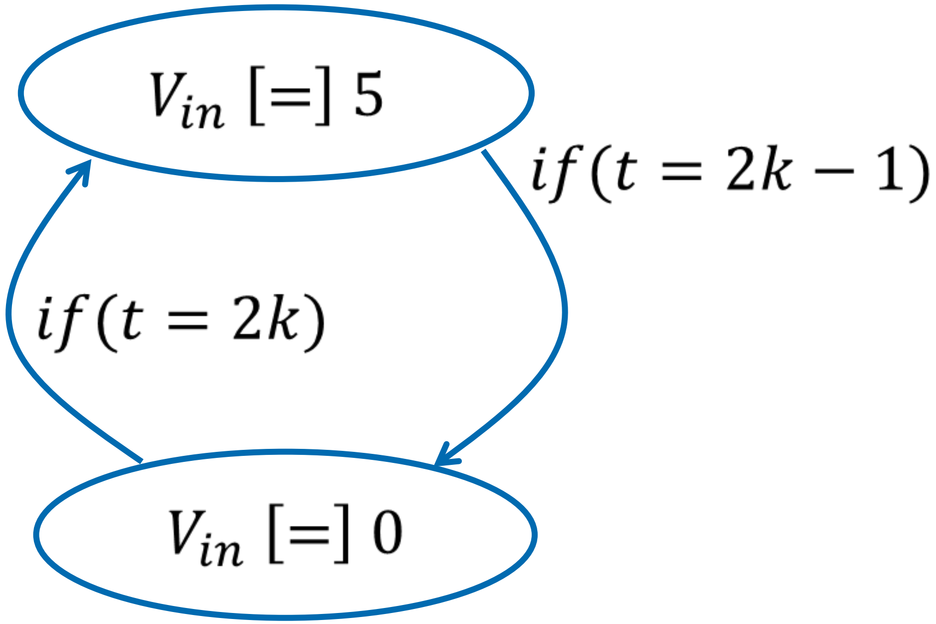
RC circuit

For $k = 1, 2, \ldots$



$$\dot{V}_c [=] \frac{V_{in} - V_c}{RC}$$

Mode 1: charge



Mode 2: discharge

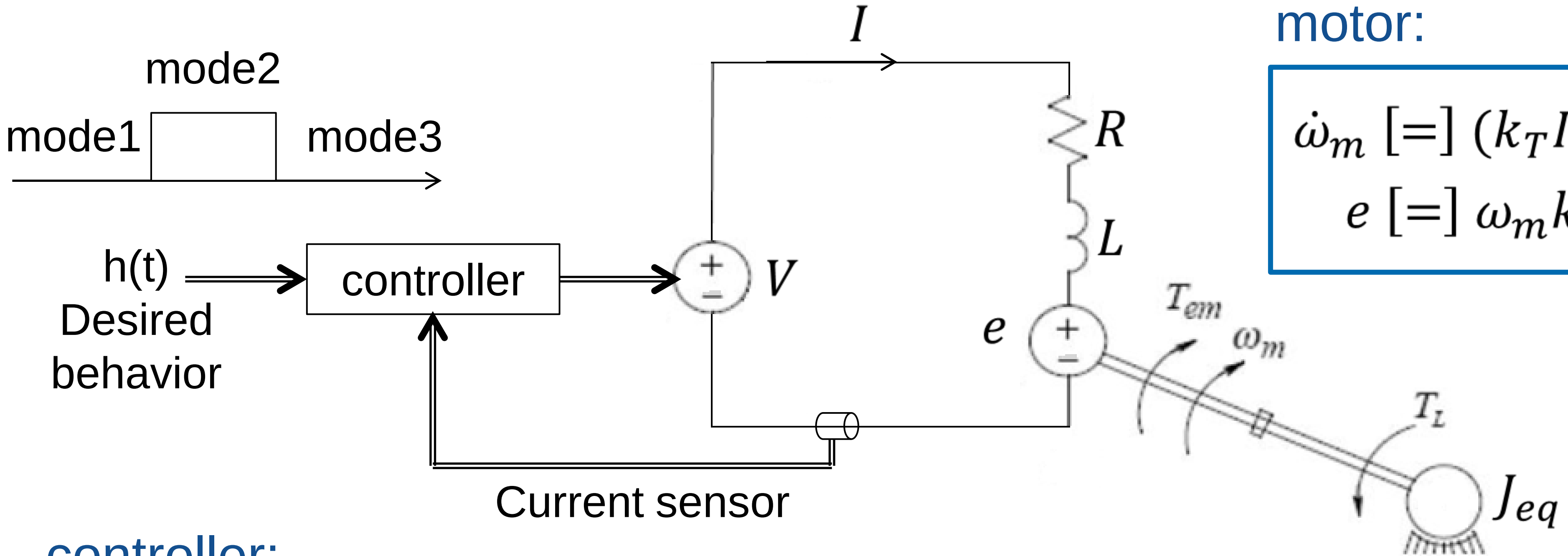
Electric motor

RL circuit: $\dot{I} \; [=] \; (V - e - RI)/L$

motor:

$$\dot{\omega}_m \; [=] \; (k_T I - T_L)/J_{eq}$$

$$e \; [=] \; \omega_m k_E$$



controller:

$$V \; [=] \; k_p(h - I) + k_i \int (h - I)\, dt$$

Electric motor

RL circuit:

$$\dot{I}\ [=]\ (V - e - RI)/L$$

motor:

$$\dot{\omega}_m\ [=]\ (k_T I - T_L)/J_{eq}$$
$$e\ [=]\ \omega_m k_E$$

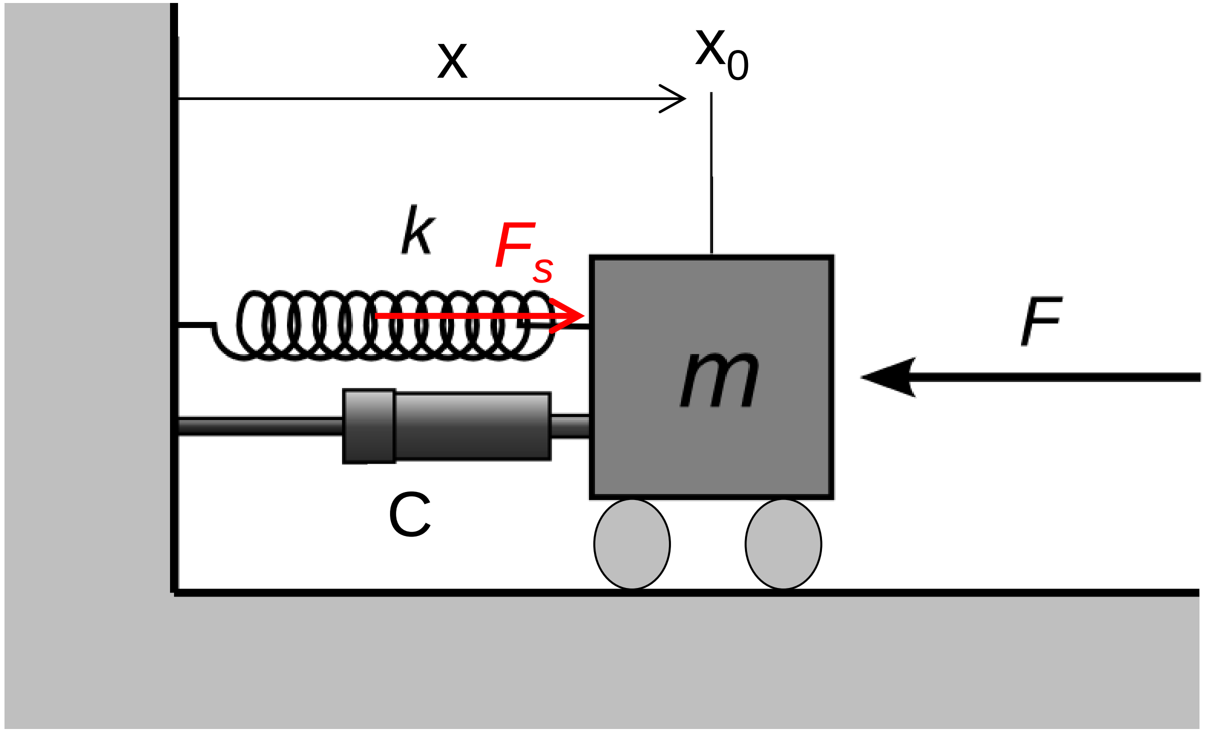
controller:

$$V\ [=]\ k_p(h - I) + k_i \int (h - I)\, dt$$

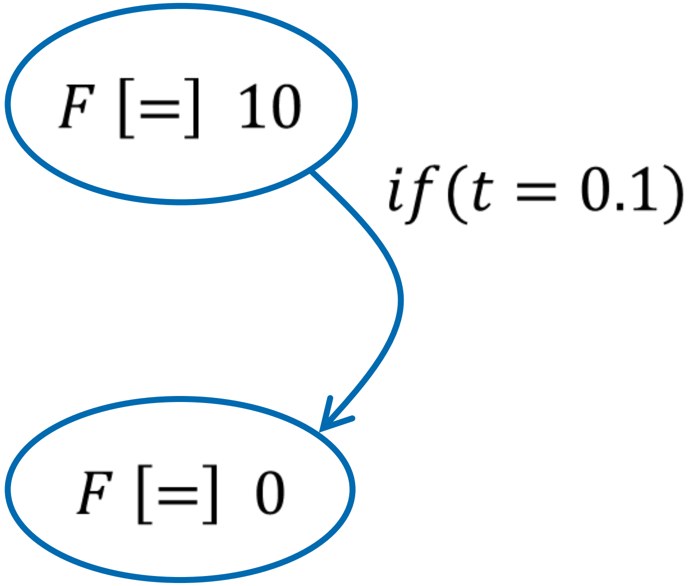
Connecting the classes:

circuit.v	[=]	controller.v
circuit.e	[=]	motor.e
motor.i	[=]	circuit.i
controller.i	[=]	circuit.i

Spring-mass system



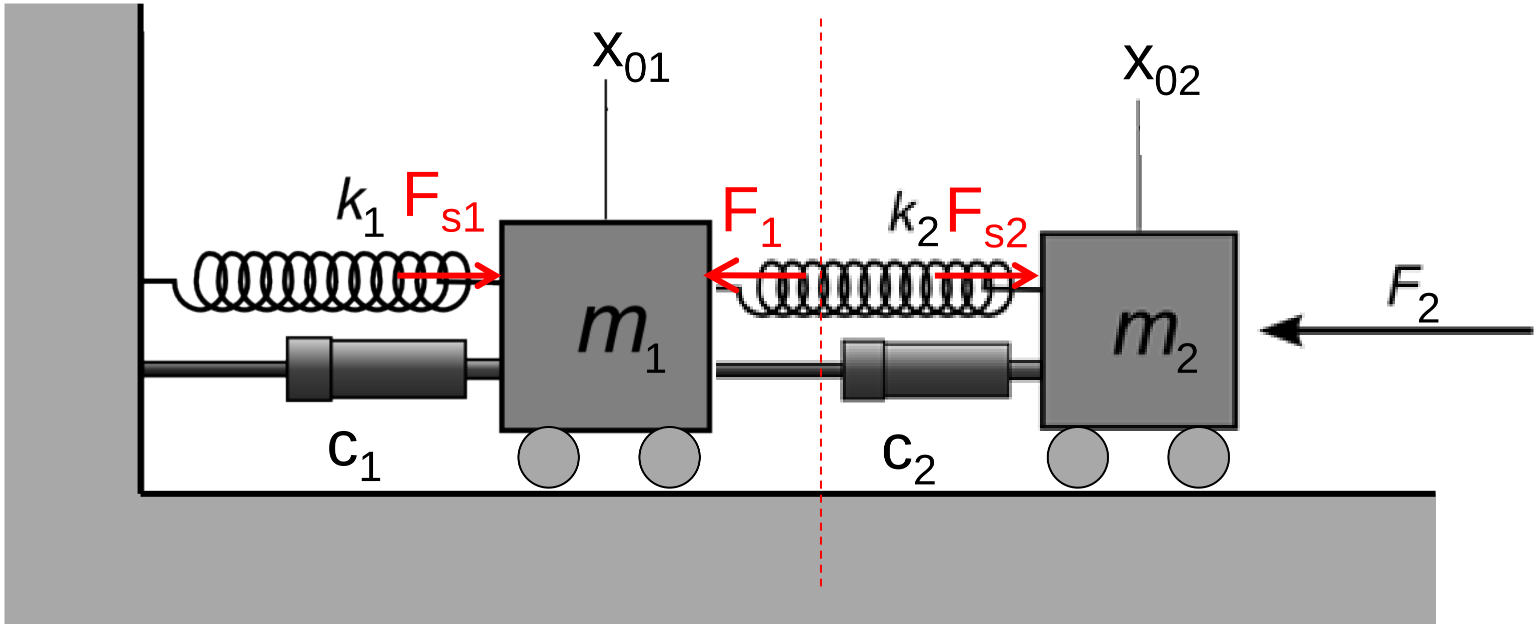
Mode 1: push



Mode 2: free motion

$$F_s \,[=]\, k(x_0 - x) - c\dot{x}$$
$$\ddot{x} \,[=]\, (F_s - F)/m$$

Two spring-mass systems

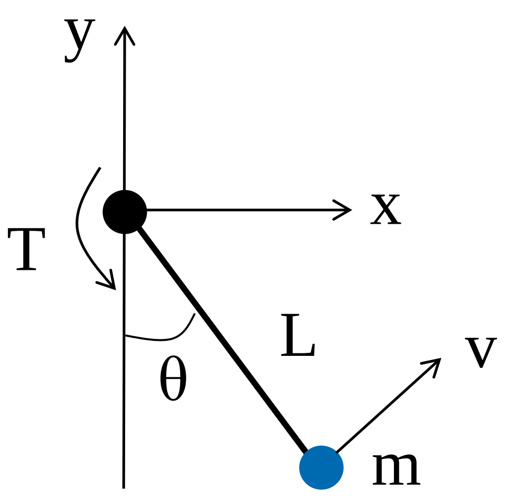


System1 System2

Connecting the two systems:

system1.F [=] system2.Fs
system2.x_0 [=] x_{02} + system1.x – x_{01}

Pendulum



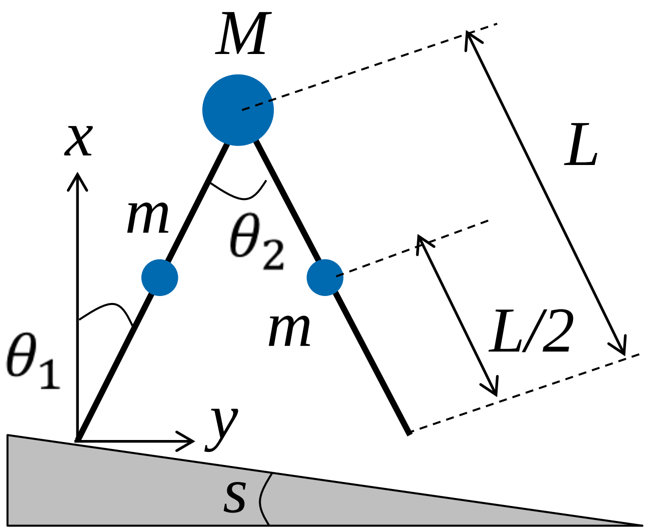
$$L = \frac{1}{2}\left(mL^2\dot{\theta}^2\right) - mgL(1 - \cos\theta)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = T$$

$$y \;[=]\; -L\cos\theta$$
$$x \;[=]\; L\sin\theta$$

$$\ddot{\theta} \;[=]\; (T - mgL\sin\theta)/(mL^2)$$
$$T \;[=]\; -k\dot{\theta}$$

Compass gait robot



Mode: swing

$$\ddot{\theta}_1 \; [=] \; f(\theta_1, \theta_2, m, L)$$
$$\ddot{\theta}_2 \; [=] \; f(\theta_1, \theta_2, m, L)$$

Guard

$$if\,(h \leq 0)\;\&\;\left(\dot{h} \leq 0\right)\;\&\;(\theta_2 > 0.05)$$

Reset map

$$\theta_1{}^{+} = \theta_1 + \theta_2$$
$$\theta_2{}^{+} = -\theta_2$$
$$\dot{\theta}_1{}^{+} = f(\theta_1, \theta_2, m, L)$$
$$\dot{\theta}_2{}^{+} = f(\theta_1, \theta_2, m, L)$$

$$h \; [=] \; 2L\sec(s)\sin\left(\frac{\theta_2}{2}\right)\sin\left(\theta_1 + \frac{\theta_2}{2} + s\right)$$

Future work

- Model smart micro-grid
  - o Physical layer
  - o Sensor network
  - o Management