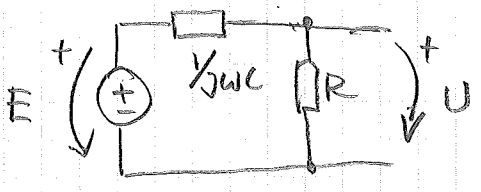
9.12



$$\begin{cases} E = 10{,}0\, e^{j\omega t} \text{ V} \\ \omega = 800 \text{ rad/s} \\ R = 1{,}25\,\text{k}\Omega \\ C = 500\,\text{nF} \end{cases}$$

Beräkna: $u(t) = |U| \sin(\omega t + \alpha)$

$$U = \frac{R}{R + \frac{1}{j\omega C}} \cdot E$$

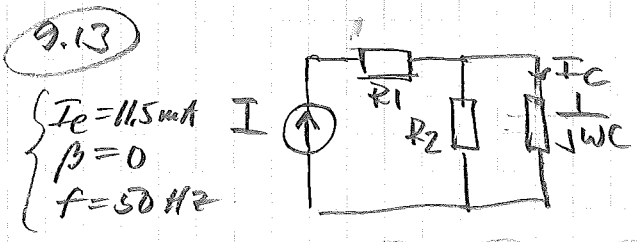
$$= \frac{1{,}25 \cdot 10^3}{(1{,}25 \cdot 10^3 - j\,2500)} \cdot 10{,}0\, e^{j\omega t}$$

$$= \frac{1{,}25 \cdot 10{,}0}{(1{,}25 - j\,2{,}5)} e^{j\omega t} = \frac{12{,}5}{2{,}795\, e^{-j63{,}4°}} e^{j\omega t}$$

$$= 4{,}47\, e^{j(\omega t + 63{,}4°)} \text{ V}$$

$$u(t) = \text{Im}\{U\} = 4{,}47 \sin(\omega t + 63{,}4°) \text{ V}$$

9.13



$$\begin{cases} I_e = 11{,}5\,\text{mA} \\ \beta = 0 \\ f = 50\,\text{Hz} \end{cases}$$

$R_1 = 3{,}3\,\text{k}\Omega$; $R_2 = 2{,}3\,\text{k}\Omega$

$C = 1{,}0\,\mu\text{F}$

Bestäm $i_c(t) = |I_c| \sin(\omega t + \beta)$

$$I_c = \frac{R_2}{R_2 + \frac{1}{j\omega C}} \cdot I \quad ; \quad i(t) = I_e \cdot \sqrt{2} \sin(2\pi 50\, t) \quad (\omega)$$

$$I = I_e \cdot \sqrt{2}\, e^{j\omega t}$$

$$= \frac{2{,}3 \cdot 10^3}{2{,}3 \cdot 10^3 - j\frac{1}{2\pi 50 \cdot 1 \cdot 10^{-6}}} \underbrace{I_e \cdot \sqrt{2}}_{16{,}26\,\text{mA}}\, e^{j\omega t}$$

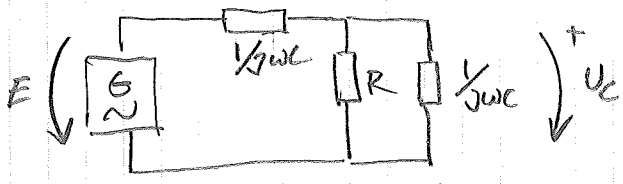
$$= \frac{2{,}3 \cdot 10^3}{2{,}3 \cdot 10^3 - j\,3{,}18 \cdot 10^3} \cdot 16{,}26\, e^{j\omega t}$$

$$= \frac{2{,}3}{2{,}3 - j\,3{,}18} \cdot 16{,}26 \cdot e^{j\omega t} = \frac{2{,}3 \cdot 16{,}26}{3{,}93 \cdot e^{-j54{,}1°}} e^{j\omega t}$$

$$= 9{,}54\, e^{j(\omega t + 54{,}1°)} \text{ mA}$$

$$i_c(t) = \text{Im}\{I_c\} = 9{,}54 \sin(\omega t + 54{,}1°) \text{ mA}$$

9.14



$E = 600\, e^{j(\omega t + 36°)}$ mV ; $\omega = 5000$ rad/s

$R = 10\Omega$; $C = 20\mu F$

Berāķina $U_c(t)$

$$U_c = \frac{R \,//\, \frac{1}{j\omega C}}{R \,//\, \frac{1}{j\omega C} + \frac{1}{j\omega C}} \cdot E$$

$$\times \quad R \,//\, \frac{1}{j\omega C} = \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \quad \begin{matrix} \times j\omega C \\ \times j\omega C \end{matrix} = \frac{R}{1 + j\omega RC}$$

$$\times \quad \frac{\frac{R}{(1+j\omega RC)}}{\frac{R}{(1+j\omega RC)} + \frac{1}{j\omega C}} \quad \begin{matrix} \times (1+j\omega RC) \\ \times (1+j\omega RC) \end{matrix} = \frac{R}{R + \frac{(1+j\omega RC)}{j\omega C}}$$

$$= \frac{R}{R + \frac{1}{j\omega C} + R} = \frac{R}{2R - j\frac{1}{\omega C}}$$

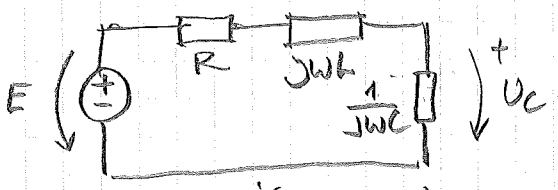
$$U_c = \frac{R}{2R - j\frac{1}{\omega C}} \cdot E = \frac{10}{\left(20 - j\frac{1}{5\cdot 10^3 \cdot 20\cdot 10^{-6}}\right)} \cdot 600 e^{j36°} e^{j\omega t}$$

$$= \underbrace{\frac{10}{(20 - j10)}}_{\frac{1}{(2-j)}} 600 e^{j36°} e^{j\omega t}$$

$$= \frac{600 \cdot e^{j36°}}{2{,}24 \cdot e^{j26{,}6°}} e^{j\omega t} = 267{,}9\, e^{j(\omega t + 62{,}6°)} \text{ mV}$$

$$u_c(t) = Im\{U_c\} = 267{,}9 \sin(\omega t + 62{,}6°) \text{ mV}$$

9.15



$R = 10\,\Omega$
$L = 10\,mH$; $C = 5{,}0\,\mu F$

$E = 600\, e^{j(\omega t + 36{,}0°)}$ mV ; $\omega = 5000$ rad/s

---

Bestäm $u_C(t)$

$$U_C = \frac{1/j\omega C}{1/j\omega C + R + j\omega L} \cdot E \quad \left(\times j\omega C\right) = \frac{1}{1 + j\omega RC - \omega^2 LC} \cdot E$$

$$= \frac{1}{(1-\omega^2 LC) + j\omega RC} \cdot E = \frac{1}{-0{,}25 + j0{,}25} \cdot E$$

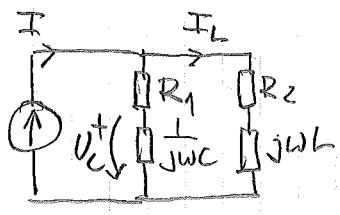
$1 - 25\cdot 10^6 \cdot 10\cdot 10^{-3} \cdot 5\cdot 10^{-6}$
$= 1 - 1{,}25 = -0{,}25$

$5\cdot 10^{-3} \cdot 10 \cdot 5\cdot 10^{-6}$
$250\cdot 10^{-3}$

$$= \frac{0{,}6 \cdot e^{j36{,}0°}}{0{,}354\, e^{j135°}} \; e^{j\omega t} = 1{,}7\, e^{j(\omega t - 99{,}0°)} \quad \checkmark$$

$$u_C(t) = \mathrm{Im}\{U_C\} = 1{,}7 \sin(\omega t - 99{,}0°)\ V$$

(9.16)



$R_1 = 30\,\Omega$; $R_2 = 10\,\Omega$

$C = 6{,}25\,\mu F$; $L = 17{,}5\,mH$

$I = 0{,}8\, e^{j\omega t}$; $\omega = 4000\ rad/s$

---

a) Beräkna strömmen $i_L(t)$

$$I_L = \frac{(R_1 + \frac{1}{j\omega C})}{(R_1 + \frac{1}{j\omega C}) + (R_2 + j\omega L)} \cdot I$$

$$= \frac{(R_1 + \frac{1}{j\omega C})}{(R_1 + R_2) + j(\omega L - \frac{1}{\omega C})} \cdot I = \frac{(30 - j40)}{40 + j(70-40)} \cdot I$$

$$= \frac{(3 - j4)}{(4 + j3)}\ 0.8\ e^{j\omega t} = \frac{5 \cdot e^{-j53{,}1°}}{5 \cdot e^{j36{,}9°}}\ 0.8\ e^{j\omega t}$$

$$= 0.8\, e^{j(\omega t - 90°)} \quad A$$

$$i_L(t) = Im\{I_L\} = 0{,}8 \sin(\omega t - 90°) \quad A$$

---

b) Beräkna spänningen $u_C(t)$

$$U_C = (I - I_L) \cdot \frac{1}{j\omega C}$$

$$= \left(0.8\, e^{j\omega t} - 0.8\, e^{j(\omega t - 90°)}\right) \cdot \frac{1}{j\omega C}$$

$$= \left(0.8 - 0.8\, e^{-j90°}\right) e^{j\omega t} \cdot \frac{1}{j\omega C}$$

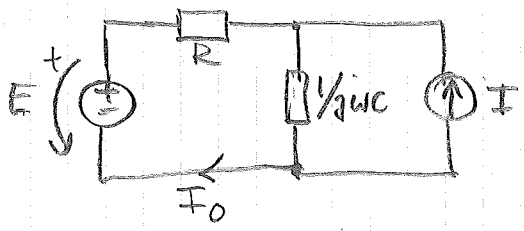
$$= (0.8 + j0.8)\, \frac{1}{j\omega C}\, e^{j\omega t}$$

$$= \frac{(0.8 + j0.8)}{j\,25 \cdot 10^{-3}}\, e^{j\omega t} = \frac{1{,}13 \cdot e^{j45°}}{0{,}025 \cdot e^{j90°}} \cdot e^{j\omega t}$$

$$= 45{,}2\, e^{j(\omega t - 45°)} \quad V$$

$$u_C(t) = 45{,}2 \sin(\omega t - 45°) \quad V$$

(9.17)



$R = 10\,\Omega$
$C = 10\,\mu F$

$E = 25\,e^{j\omega t}$ V ; $I = 2{,}5\,e^{j\omega t}$ A ; $\omega = 10$ krad/s.

a) Bestäm $i_0(t)$

$$I_0 = \frac{E}{R + \frac{1}{j\omega C}} - \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \cdot I$$

$$= \frac{E}{(10 - j10)} - \frac{-j10}{(10 - j10)} \cdot I$$

$(10 - j10) = 10(1-j)$; $\frac{-j}{1-j} = e^{-j90°}$

$$= \left(\frac{2{,}5}{\sqrt{2}\, e^{-j45°}} - \frac{e^{-j90°}}{\sqrt{2}\, e^{-j45°}} \cdot 2{,}5 \right) e^{j\omega t}$$

$$= \left(\frac{2{,}5}{\sqrt{2}} e^{j45°} - \frac{2{,}5}{\sqrt{2}} e^{-j45°} \right) e^{j\omega t}$$

$(1{,}25 + j1{,}25) - (1{,}25 - j1{,}25) = j2{,}5$

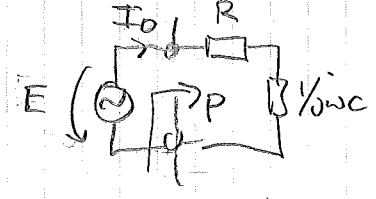
$$= 2{,}5\, e^{j90°} \cdot e^{j\omega t} = 2{,}5\, e^{j(\omega t + 90°)} \text{ A}$$

$i_0(t) = \mathrm{Im}\{I_0\} = 2{,}5 \sin(\omega t + 90°)$ A

b) Effektförbrukningen i R.

$$P = R \cdot I_{0e}^2 = 10 \cdot \left(2{,}5/\sqrt{2}\right)^2 = 31{,}25 \text{ W}$$

c)



$P_E = E \cdot I_0 \cdot \cos(\varphi)$; $\varphi = -90°$

$E = 25\, e^{j\omega t}$

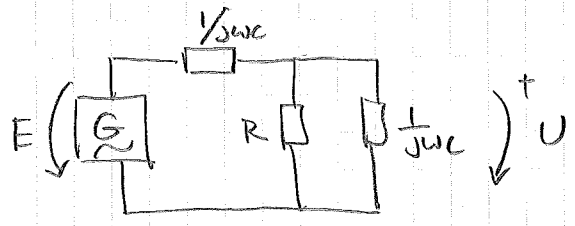
$I_0 = 2{,}5\, e^{j(\omega t + 90°)}$ $\Rightarrow P_E = 0$

All aktiv effekt mäste komma från I

$P_I = R \cdot (I_{0e})^2$

$= 10 \cdot \left(\frac{2{,}5}{\sqrt{2}}\right)^2 = 31{,}25$ W

9.18



$$U = \frac{R /\!/ \frac{1}{j\omega C}}{R /\!/ \frac{1}{j\omega C} + \frac{1}{j\omega C}} \cdot E$$

där $R /\!/ \frac{1}{j\omega C} = \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC}$

$$\Rightarrow U = \frac{\frac{R}{1+j\omega RC}}{\frac{R}{1+j\omega RC} + \frac{1}{j\omega C}} \cdot E = \frac{\frac{j\omega RC}{1+j\omega RC}}{\frac{j\omega RC}{1+j\omega RC} + 1} \cdot E$$

$$= \frac{j\omega RC}{j\omega RC + 1(1+j\omega RC)} \cdot E = \frac{j\omega RC}{1 + j2\omega RC} \cdot E$$

$$F(\omega) = \frac{U}{E} = \frac{j\omega RC}{(1+j2\omega RC)} = \underbrace{\frac{|U|}{|E|}}_{|F(\omega)|} e^{\underbrace{j(90° - \arctan(2\omega RC))}_{\text{Arg } F(\omega)}}$$

a) Vid vilkens frekvens ligger u 45° före generatorspänningen e?

$\Rightarrow \text{Arg}\{F(\omega)\} = 45° \Rightarrow 90° - \arctan(2\omega RC) = 45°$

$\arctan(2\omega RC) = 45°$

$2\omega RC = 1 \Rightarrow \omega = \frac{1}{2RC}$ //

---

b) Vid vilken frekvens är $U_e = 0{,}25 \cdot E_e$

$$|F(\omega)| = \frac{|U|}{|E|} = \frac{U_e}{E_e} = 0{,}25$$

$$\Rightarrow \frac{|j\omega RC|}{|1 + j2\omega RC|} = \frac{1}{4}$$

$$\frac{\omega RC}{\sqrt{1^2 + (2\omega RC)^2}} = \frac{1}{4}$$

$$\frac{(\omega RC)^2}{1^2 + (2\omega RC)^2} = \frac{1}{16}$$

$$16(\omega RC)^2 = 1 + 4(\omega RC)^2$$

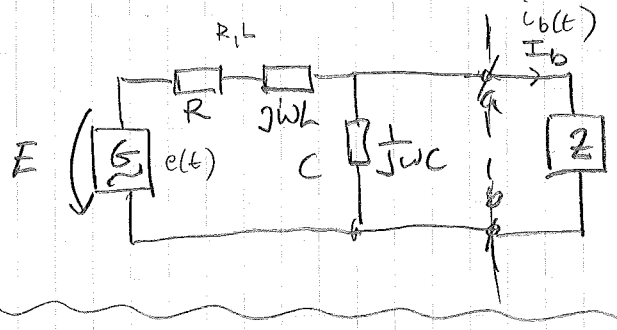
$$12\omega^2(RC)^2 = 1$$

$$\omega^2 = \frac{1}{12}\frac{1}{(RC)^2}$$

$$\omega = \frac{1}{2\sqrt{3}RC}$$ //

(9.19)

Komplett schema



$E = 16\,e^{j\omega t}$ V, $\omega = 100$ krad/s

$R = 4\,\Omega$, $L = 30\,\mu H$

$C = 1{,}667\,\mu F$

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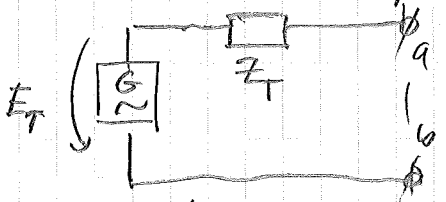
a) Ekvivalent tvåpol:



$$E_T = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + (R + j\omega L)} \cdot E \quad \begin{matrix} \cdot j\omega C \\ \cdot j\omega C \end{matrix} = \frac{1}{(1 - \omega^2 LC + j\omega RC)} \cdot E$$

$$= \frac{1}{(1 - \underbrace{10^6 \cdot 30 \cdot 10^{-6} \cdot 1{,}667 \cdot 10^{-6}}_{\approx 0{,}5}) + j\underbrace{10^3 \cdot 4 \cdot 1{,}667 \cdot 10^{-6}}_{0{,}667}} \cdot E = \frac{1}{0{,}5 + j\,0{,}667} \cdot E$$

$$= \frac{1 \cdot e^{j0^\circ}}{0{,}834 \cdot e^{j53{,}1^\circ}} \cdot 16 \cdot e^{j\omega t} = \frac{16 \cdot 1}{0{,}834}\, e^{j(\omega t - 53{,}1^\circ)} = 19{,}2\, e^{j(\omega t - 53{,}1^\circ)}\ \text{V}$$

$$Z_T = (R + j\omega L) /\!/ \frac{1}{j\omega C} = \frac{(R + j\omega L) \cdot \frac{1}{j\omega C}}{(R + j\omega L) + \frac{1}{j\omega C}} \quad \begin{matrix} \\ \cdot j\omega C \end{matrix}$$

$$= \frac{(R + j\omega L)}{(1 - \omega^2 LC) + j\omega RC} = \frac{4 + j\,10^3 \cdot 30 \cdot 10^{-6}}{(1 - \underbrace{10^6 \cdot 30 \cdot 10^{-6} \cdot 1{,}667 \cdot 10^{-6}}_{\approx 0{,}5}) + j\underbrace{10^3 \cdot 4 \cdot 1{,}667 \cdot 10^{-6}}_{0{,}667}}$$

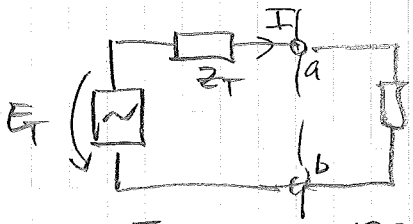
$$= \frac{4 + j3}{0{,}5 + j\,0{,}667} = \frac{5 \cdot e^{j36{,}9^\circ}}{0{,}834\, e^{j53{,}1^\circ}} \approx 6{,}0\, e^{-j16{,}2^\circ}$$

$$\approx (5{,}76 - j\,1{,}67)\ \Omega$$

Suboptimalt om: $R = |Z_T| \approx 6\,\Omega \Rightarrow P \approx 7{,}8$ W

b) Lasten $Z = R = 2\,\Omega$.

Bestäm $i(t)$ och P i lasten då $Z = R = 2\,\Omega$.



$P = R \cdot I_e^2$

$P = R \cdot I_e^2$

$$I = \frac{E_T}{(Z_T + R)} = \frac{19{,}2 \cdot e^{j(\omega t - 53{,}1^\circ)}}{(5{,}76 - j\,1{,}67) + 2} = \frac{19{,}2\, e^{-j53{,}1^\circ}\, e^{j\omega t}}{7{,}94 \cdot e^{-j12{,}1^\circ}} \approx 2{,}42\, e^{j(\omega t - 41^\circ)}\ \text{A}$$

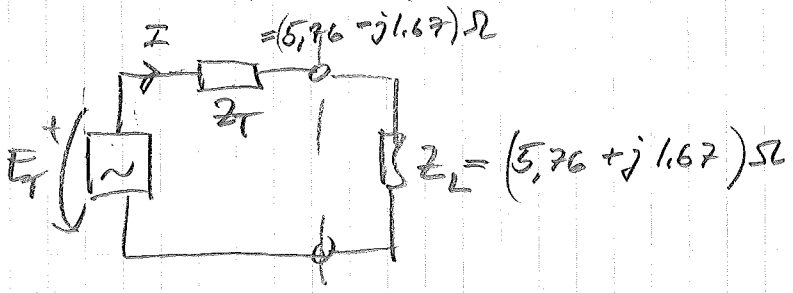
$$i(t) = \text{Im}\{I\} = 2{,}42 \sin(\omega t - 41^\circ)\ \text{A}$$

$$P = R \cdot I_e^2 = 2 \cdot \left(\frac{2{,}42}{\sqrt{2}}\right)^2 = (2{,}42)^2 \approx 5{,}86\ \text{W}$$

(9.19) forts
c)

Z_L kan väljas fritt.
Bestäm Z_L för maximal effektutv. P
Bestäm P.

$$Z_L = \overline{Z_T} = \overline{(5{,}76 - j1{,}67)} = (5{,}76 + j1{,}67)\,\Omega$$



$$\boxed{P = R_L \cdot I_e^2}$$

$$\begin{cases} R_L = 5{,}76\,\Omega \\ I = \dfrac{E_T}{(Z_T + Z_L)} = \dfrac{E_T}{2 \cdot 5{,}76} = \dfrac{19{,}2 \cdot e^{j(\omega t - 53{,}7°)}}{2 \cdot 5{,}76} \\ \approx 1{,}67\, e^{j(\omega t - 53{,}7°)} \quad A \\ \text{Således} \quad I_e = 1{,}67/\sqrt{2}\ A \end{cases}$$

$$\Rightarrow P = R_L \cdot I_e^2 = 5{,}76 \cdot \left(\frac{1{,}67}{\sqrt{2}}\right)^2 \approx 8\,W$$