Exam in Sensor system, 7.5 credits.
Course code: et2009
Date: 2010-05-24

Allowed items on the exam:
Tables of Mathematical formulas.
Calculator.

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Maximum points: 28.
In order to pass the examination with a grade 3 a minimum of 12 points is required.
To get a grade 4 a minimum of 17 points is required, and to get a grade 5 a minimum of 23 points is required.

Give your answer in a readable way and motivate your assumptions.

Good Luck!

Questions of 2 points.

1.
A force sensor has an input range of 0 to 10 kN and an output range of 0 to 5 V at a standard temperature of 20 ˚C. At 30 ˚C the output range is 0 to 5.5 V.
Quantify the environmental effect, i.e. find K_M and K_I.

2.
A level measurement system consists of three ideal linear elements in series with sensitivities of K_1=0.050, K_2=21.5 and K_3=0.99. Find the system error for a true value input of 5.0 metres.

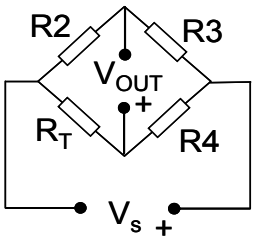
3.
A potentiometer has a total length of 10 cm and a resistance of 100 Ω.
a) Calculate the supply voltage so that the power dissipation = 1 W.
b) Draw the Thevenin equivalent circuit for 7 cm displacement.
c) The potentiometer is connected to a recorder with a resistance R_L. Find R_L such that the recorder voltage is 5% less than the open circuit voltage at 7 cm displacement.

4.
A strain gauge having an unstrained resistance of 120 Ω and a gauge factor of 2.1 is bounded onto a steel beam so that it experiences a tensile stress of 10^8 Pa.
If Young's modulus for steel is 2 x 10^{11} Pa, calculate the strained resistance of the gauge.
The relation between stress (σ) and strain (e) is: σ=E e, where E is the Young modulus.

5.
A force sensor has a steady state sensitivity of 10^{-6} mN^{-1}.
If the force input has a range of 0 to 5 kN, find the corresponding displacement output range.

6.
The voltage of a type T thermocouple is measured to be 8.561 mV relative to a reference junction temperature of 20 ˚C. Use the table (reference junction at 0˚C, included in this exam) to find the temperature of the measured junction.

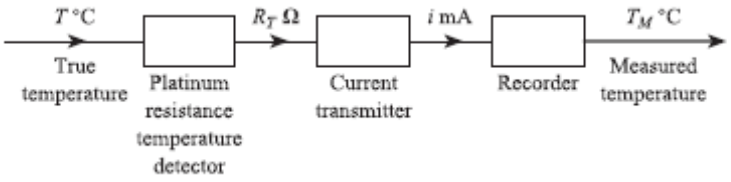
7.
A platinum resistance sensor has a resistance $R_T = R_0(1+\alpha T)$, where R_0=100 Ω
and α=4 x 10^{-3} $^\circ C^{-1}$.
a) The above sensor is incorporated into a bridge circuit which has R_3/R_2=100. Find the value of R_4 such that V_{OUT}=0 V at 0 ˚C.
b) Complete the bridge design by calculating the supply voltage required to give V_{OUT}=100 mV at 100 ˚C.



8.
An AD-converter has an input range of [0, 5] V and incorporates a 10-bit binary encoder.
Find the maximum quantisation error.

Questions of 4 points.

9.
A temperature measurement system consists of a platinum resistance temperature detector, current transmitter and recorder. The model equations, and parameters for each element are presented below. Assuming that all probability distributions are normal, calculate the mean and standard deviation for the output and for the error when the true temperature is T=120 ˚C.

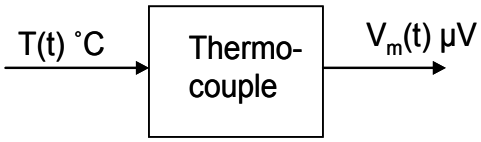


Platinum resistance temperature detector:
Model equation: $R_T = R_0\left(1 + \alpha T + \beta T^2\right)$ [Ω].
Mean values: R_0=100.0 Ω, α=3.909 x 10^{-3}, β= -5.897 x 10^{-7}.
Standard deviations: σ_{R0}=4.33 x 10^{-2}, σ_α=0.0, σ_β=0.0.

Current transmitter:
4 to 20 mA output for 138.5 to 149.8 Ω input (100 to 130 ˚C).
ΔT_a = deviation of ambient temperature from 20 ˚C.
Model equation: $i=KR_T + K_M R_T \Delta T_a + K_I \Delta T_a + a$ [mA].
Mean values: K=1.4134, K_M=1.4134 x 10^{-4}, K_I= -1.637 x 10^{-2}, a= -191.76, ΔT_a= -10.
Standard deviations: σ_a=0.24, $\sigma_{\Delta Ta}$=6.7, σ_K=0.0, σ_{KM}=0.0, σ_{KI}=0.0.

Recorder:
Model equation: $T_M=Ki + b$ [˚C].
Mean values: K=1.875, b=92.50.
Standard deviations: σ_b=0.10, σ_K=0.0.

10.
A thermocouple is used to measure the temperature inside a vessel, which is a part of a high-speed batch process. At time t=0, with the vessel at an initial temperature of 50 ˚C, the vessel is instantaneously filled with gas at 150 ˚C. One minute later, instantaneously the gas is removed and the vessel is filled with liquid at 50 ˚C.



The thermocouple can be regarded as having linear steady-state characteristics and first-order dynamics.

Data:
Transfer function of the thermocouple: $G(s) = \dfrac{K}{(1 + \varpi s)}$
Thermocouple sensitivity K= 40 μV˚C^{-1}.
Time constant gas τ_g= 1 s.
Time constant liquid τ_l= 0.2 s.

a) The input is a step of height A applied at t=0. Show that the output is $V_m(t) = AK\left(1 - e^{-\frac{t}{\tau}}\right)$ for t≥0

by using the Laplace transform (table is included in this exam).
b) Sketch, in one graph, the input T(t) and the output $V_m(t)$. The axes of the graph should have suitable scales and the answer should include supporting numerical calculations for t=1, 5, 10, 61 and 65 s.
c) The following elements, amplifier and recorder, has a steady state sensitivity of $K_A=10^3$ respectively $K_R= 25\ ^\circ C\ V^{-1}$. Calculate the dynamic error at the t-values given in b) if the dynamics of the system are determined by the thermocouple only.

11.
The histogram below is computed from 2000 outputs from a measurement system in mV, when the input is at a constant value and the noise is random and normal distributed N(0,σ).
In the figure the two columns, from left to right, are the bin centres and the number of measurements in each bin.

a) Use the histogram to estimate the mean value (=m) and standard deviation (=σ) of the output and present the measurement as m±σ mV.
Hint: the N(0,1) table can be used in the estimation of σ.
b) Estimate the Signal-to-Noise Ratio (SNR).
c) The SNR should be improved by a factor of 10 due to averaging. Find the minimum number of signals that must be used in the averaging to get this improvement in the SNR.
If no result from a) is given, you are free to assume values of m and σ when solving b) and c).

599.3554	2
601.4991	3
603.6429	16
605.7867	20
607.9305	40
610.0743	70
612.2180	113
614.3618	176
616.5056	219
618.6494	283
620.7932	279
622.9369	265
625.0807	187
627.2245	144
629.3683	85
631.5121	41
633.6559	38
635.7996	13
637.9434	3
640.0872	3

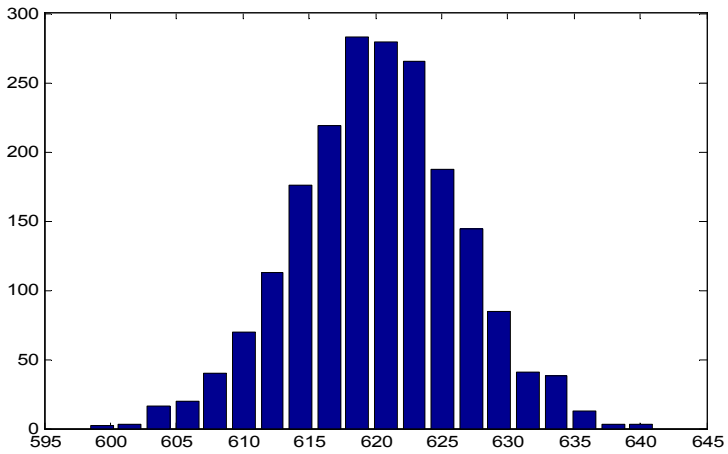
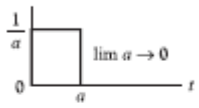
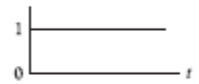
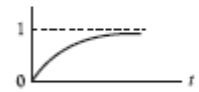
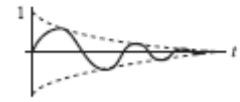
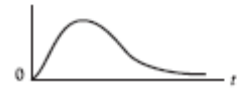


Table Prob. 22 e.m.f. in mV for type T thermocouple (measured junction at T °C, reference junction at 0 °C).

Temp. °C	0	1	2	3	4	5	6	7	8	9	10
0	0.000	0.039	0.078	0.117	0.156	0.195	0.234	0.273	0.312	0.351	0.391
10	0.391	0.430	0.470	0.510	0.549	0.589	0.629	0.669	0.709	0.749	0.789
20	0.789	0.830	0.870	0.911	0.951	0.992	1.032	1.073	1.114	1.155	1.196
30	1.196	1.237	1.279	1.320	1.361	1.403	1.444	1.486	1.528	1.569	1.611
40	1.611	1.653	1.695	1.738	1.780	1.822	1.865	1.907	1.950	1.992	2.035
50	2.035	2.078	2.121	2.164	2.207	2.250	2.294	2.337	2.380	2.424	2.467
60	2.467	2.511	2.555	2.599	2.643	2.687	2.731	2.775	2.819	2.864	2.908
70	2.908	2.953	2.997	3.042	3.087	3.131	3.176	3.221	3.266	3.312	3.357
80	3.357	3.402	3.447	3.493	3.538	3.584	3.630	3.676	3.721	3.767	3.813
90	3.813	3.859	3.906	3.952	3.998	4.044	4.091	4.137	4.184	4.231	4.277
100	4.277	4.324	4.371	4.418	4.465	4.512	4.559	4.607	4.654	4.701	4.749
110	4.749	4.796	4.844	4.891	4.939	4.987	5.035	5.083	5.131	5.179	5.227
120	5.227	5.275	5.324	5.372	5.420	5.469	5.517	5.566	5.615	5.663	5.712
130	5.712	5.761	5.810	5.859	5.908	5.957	6.007	6.056	6.105	6.155	6.204
140	6.204	6.254	6.303	6.353	6.403	6.452	6.502	6.552	6.602	6.652	6.702
150	6.702	6.753	6.803	6.853	6.903	6.954	7.004	7.055	7.106	7.156	7.207
160	7.207	7.258	7.309	7.360	7.411	7.462	7.513	7.564	7.615	7.666	7.718
170	7.718	7.769	7.821	7.872	7.924	7.975	8.027	8.079	8.131	8.183	8.235
180	8.235	8.287	8.339	8.391	8.443	8.495	8.548	8.600	8.652	8.705	8.757
190	8.757	8.810	8.863	8.915	8.968	9.021	9.074	9.127	9.180	9.233	9.286
200	9.286	9.339	9.392	9.446	9.499	9.553	9.606	9.659	9.713	9.767	9.820
210	9.820	9.874	9.928	9.982	10.036	10.090	10.144	10.198	10.252	10.306	10.360
220	10.360	10.414	10.469	10.523	10.578	10.632	10.687	10.741	10.796	10.851	10.905
230	10.905	10.960	11.015	11.070	11.125	11.180	11.235	11.290	11.345	11.401	11.456
240	11.456	11.511	11.566	11.622	11.677	11.733	11.788	11.844	11.900	11.956	12.011
250	12.011	12.067	12.123	12.179	12.235	12.291	12.347	12.403	12.459	12.515	12.572
260	12.572	12.628	12.684	12.741	12.797	12.854	12.910	12.967	13.024	13.080	13.137
270	13.137	13.194	13.251	13.307	13.364	13.421	13.478	13.535	13.592	13.650	13.707
280	13.707	13.764	13.821	13.879	13.936	13.993	14.051	14.108	14.166	14.223	14.281
290	14.281	14.339	14.396	14.454	14.512	14.570	14.628	14.686	14.744	14.802	14.860
300	14.860	14.918	14.976	15.034	15.092	15.151	15.209	15.267	15.326	15.384	15.443
310	15.443	15.501	15.560	15.619	15.677	15.736	15.795	15.853	15.912	15.971	16.030
320	16.030	16.089	16.148	16.207	16.266	16.325	16.384	16.444	16.503	16.562	16.621
330	16.621	16.681	16.740	16.800	16.859	16.919	16.978	17.038	17.097	17.157	17.217
340	17.217	17.277	17.336	17.396	17.456	17.516	17.576	17.636	17.696	17.756	17.816

Table 4.1 Laplace transforms of common time functions $f(t)$.[a]

$$\mathcal{L}[f(t)] = \bar{f}(s) = \int_0^\infty e^{-st} f(t)\,dt$$

Function	Symbol	Graph	Transform
1st derivative	$\frac{d}{dt}f(t)$		$s\bar{f}(s) - f(0-)$
2nd derivative	$\frac{d^2}{dt^2}f(t)$		$s^2\bar{f}(s) - sf(0-) - \dot{f}(0-)$
Unit impulse	$\delta(t)$		1
Unit step	$\mu(t)$		$\frac{1}{s}$
Exponential decay	$\exp(-\alpha t)$		$\frac{1}{s+\alpha}$
Exponential growth	$1 - \exp(-\alpha t)$		$\frac{\alpha}{s(s+\alpha)}$
Sine wave	$\sin \omega t$		$\frac{\omega}{s^2+\omega^2}$
Phase-shifted sine wave	$\sin(\omega t + \phi)$		$\frac{\omega\cos\phi + s\sin\phi}{s^2+\omega^2}$
Exponentially damped sine wave	$\exp(-\alpha t)\sin \omega t$		$\frac{\omega}{(s+\alpha)^2+\omega^2}$
Ramp with exponential decay	$t\exp(-\alpha t)$		$\frac{1}{(s+\alpha)^2}$

[a] Initial conditions are at $t = 0-$, just prior to $t = 0$.

Normal distribution $N(0,1)$

$$F(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{t^2}{2}}\,dt$$

x	0	1	2	3	4	5	6	7	8	9
0,0	0,5000	5040	5080	5120	5160	5199	5239	5279	5319	5359
0,1	5398	5438	5478	5517	5557	5596	5636	5675	5714	5753
0,2	5793	5832	5871	5910	5948	5997	6026	6064	6103	6141
0,3	6179	6217	6255	6293	6331	6368	6406	6443	6480	6517
0,4	6554	6591	6628	6664	6700	6736	6772	6808	6844	6879
0,5	6915	6950	6985	7019	7054	7088	7123	7157	7190	7224
0,6	7257	7291	7324	7357	7389	7422	7454	7486	7517	7549
0,7	7580	7611	7642	7673	7704	7734	7764	7794	7823	7852
0,8	7881	7910	77939	7967	7995	8023	8051	8078	8106	8133
0,9	8159	8186	8212	8238	8264	8289	8315	8340	8365	8389
1,0	8413	8438	8461	8485	8508	8531	8554	8577	8599	8621
1,1	8643	8665	8686	8708	8729	8749	8770	8790	8810	8830
1,2	8849	8869	8888	8907	8925	8944	8962	8980	8997	9015
1,3	9032	9049	9066	9082	9099	9115	9131	9147	9162	9177
1,4	9192	9207	9222	9236	9251	9265	9279	9292	9306	9319
1,5	9332	9345	9357	9370	9382	9394	9406	9418	9429	9441
1,6	9452	9463	9474	9484	9495	9505	9515	9525	9535	9545
1,7	9554	9564	9573	9582	9591	9599	9608	9616	9625	9633
1,8	9641	9649	9656	9664	9671	9678	9686	9693	9699	9706
1,9	9713	9719	9726	9732	9738	9744	9750	9756	9761	9767
2,0	9772	9778	9783	9788	9793	9798	9803	9808	9812	9817
2,1	9821	9826	9830	9834	9838	9842	9846	9850	9854	9857
2,2	9861	9864	9868	9871	9875	9878	9881	9884	9887	9890
2,3	9893	9896	9898	9901	9904	9906	9909	9911	9913	9916
2,4	9918	9920	9922	9925	9927	9929	9931	9932	9934	9936
2,5	9938	9940	9941	9943	9945	9946	9948	9949	9951	9952
2,6	9953	9955	9956	9957	9959	9960	9961	9962	9963	9964
2,7	9965	9966	9967	9968	9969	9970	9971	9972	9973	9974
2,8	9974	9975	9976	9977	9977	9978	9979	9979	9980	9981
2,9	9981	9982	9983	9983	9984	9984	9985	9985	9986	9986
3,0	9987	9987	9987	9988	9988	9989	9989	9989	9990	9990
3,1	9990	9991	9991	9991	9992	9992	9992	9992	9993	9993
3,2	9993	9993	9994	9994	9994	9994	9994	9995	9995	9995
3,3	9995	9995	9995	9996	9996	9996	9996	9996	9996	9997
3,4	9997	9997	9997	9997	9997	9997	9997	9997	9997	9998