9

$$e = \frac{6(l-x)\cdot F}{wt^2\cdot E} \quad ; \quad x = \frac{l}{2}$$

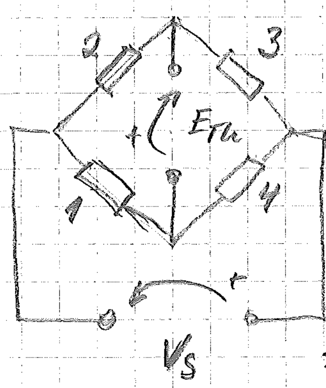
strain $e = \frac{\Delta l}{l}$

$$\Rightarrow e = \frac{6(l - l/2)\cdot F}{wt^2\cdot E} = \frac{6\cdot l\cdot \frac{1}{2}\cdot F}{wt^2\cdot E}$$

$$= \frac{6\cdot 150\cdot 10^{-3}\cdot \frac{1}{2}\cdot 250}{50\cdot 10^{-3}\cdot (3\cdot 10^{-3})^2\cdot 2\cdot 10^{11}} = \underline{\underline{12{,}5\cdot 10^{-4}}}$$

a)

$$\left.\begin{array}{l}\frac{\Delta R}{R_0} = G\cdot e\\ R = R_0 + \Delta R\end{array}\right\} \Rightarrow R = R_0(1+G\cdot e)$$



1,3 tension $+e$
$R = R_0(1+G\cdot e)$

2,4 compression $-e$
$R = R_0(1-G\cdot e)$

$$\underline{\underline{E_{Th}}} = V_S\left[\frac{R_1}{R_1+R_4} - \frac{R_2}{R_2+R_3}\right]$$

$$= V_S\left[\frac{R_0(1+G\cdot e)}{R_0(1+G\cdot e)+R_0(1-G\cdot e)} - \frac{R_0(1-G\cdot e)}{R_0(1-G\cdot e)+R_0(1+G\cdot e)}\right]$$

$$= V_S\left[\frac{(1+G\cdot e)}{2} - \frac{(1-G\cdot e)}{2}\right]$$

$$= V_S\left[\frac{1+G\cdot e-1+G\cdot e}{2}\right] = \underline{\underline{V_S\cdot G\cdot e}}$$

b) $E_{Th} = V_S\cdot G\cdot e$

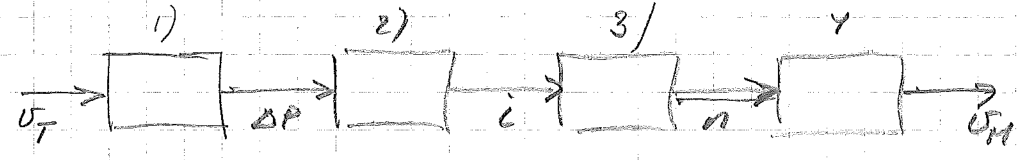
$e = 12{,}5\cdot 10^{-4}$; $G = 2{,}0$, $V_S = 15V$.

$$= 15\cdot 2.0\cdot 12{,}5\cdot 10^{-4} = \underline{\underline{37{,}5\,mV}}$$

10

$\bar{e} = +0{,}08$ m/s
$\sigma_e = 0{,}35$ m/s

3.6



1) Pitot tube $\boxed{\Delta P = \frac{1}{2} \cdot \rho \cdot v_T^2}$; $\Delta P = f(v_T)$

$\begin{cases} \bar{\rho} = 1.2 \\ h_1 = 9.8 \end{cases}$

2) Diff. pressure transmitter

$\boxed{i = K_1 \cdot \Delta P + a_1}$; $i = f(\Delta P)$

$\bar{K}_1 = 0.064$; $\bar{a}_1 = 4.0$; $h_2 = 0{,}15$

3) A/D-converter

$\boxed{n = K_2 \cdot i + a_2}$; $n = f(i)$

$\bar{K}_2 = 12{,}80$; $\bar{a}_2 = 0.0$
n = rounded off to nearest integer
$h_3 = 0.5$

4) Microcontroller

$v_M = K_3 \sqrt{n - 51}$; $v_M = f(n)$

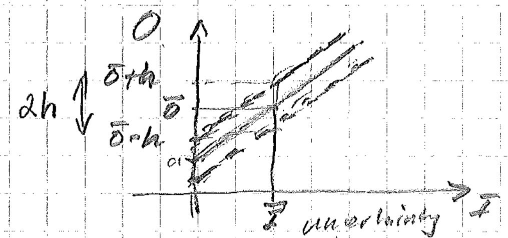
$\bar{K}_3 = 1.480$
$h_4 = 0.0$

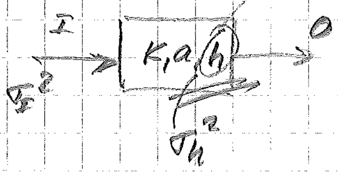
$v_T = 14.0$ m/s Estimate the mean and standard deviation of the error distr. function (Treat the rect. distr. as normal with $\sigma = h/\sqrt{3}$).

(10) cont.

Theory:

Error bands repr:

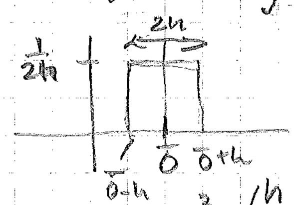




Model

$$\boxed{O = K \cdot I + a}$$

$$\left(\frac{\partial O}{\partial I}\right) = K$$



$$\sigma_h^2 = \left(\frac{h}{\sqrt{3}}\right)^2$$

uncertainty from input   uncertainty in the element

$$\Delta O = \left(\frac{\partial O}{\partial I}\right) \cdot \Delta I + \Delta h$$

"statistics"

$$\begin{cases} \sigma_O^2 = \left(\frac{\partial O}{\partial I}\right)^2 \cdot \sigma_I^2 + \sigma_h^2 = K^2 \sigma_I^2 + \sigma_h^2 \\ \bar{O} = \bar{K} \cdot \bar{I} + \bar{a} \end{cases}$$

Mean values

1) $\overline{\Delta P} = \frac{1}{2} \bar{\rho} \cdot \bar{v}_T^2 = \frac{1}{2} \cdot 1.2 \cdot (14.0)^2 = 117.6$ Pa

2) $\bar{i} = \bar{K}_1 \cdot \overline{\Delta P} + \bar{a}_1 = 0.064 \cdot 117.6 + 4.0 = 11.53$ mA

3) $\bar{n} = \bar{K}_2 \cdot \bar{i} + \bar{a}_2 = 12.80 \cdot 11.53 + 0.0 \approx 147.58 \rightarrow 148$ rounded to integer

4) $\bar{v}_M = \bar{K}_3 \cdot \sqrt{\bar{n} - 51} = 1.43 \cdot \sqrt{148 - 51} = 14.08$ m/s

$$\bar{v}_M \approx 14.08 \text{ m/s}.$$

$$\bar{e} = \bar{v}_M - v_T = 14.08 - 14.0 = \underline{\underline{0.08 \text{ m/s}}}$$

10 cont.

Standard deviation

$$\boxed{\sigma_0^2 = \left(\frac{\partial O}{\partial I}\right)^2 \sigma_I^2 + \frac{h^2}{3}}$$

1) $\sigma^2_{v_T} = 0 \Rightarrow \sigma^2_{\Delta P} = \frac{(h_1)^2}{3} = \frac{(9.8)^2}{3} = 32,0133\ (Pa)^2$

2) $$\sigma_i^2 = \underbrace{\left(\frac{\partial i}{\partial \Delta P}\right)^2 \sigma^2_{\Delta P}}_{K_1^2 \cdot \sigma^2_{\Delta P}} + \frac{(h_2)^2}{3} = \underbrace{(0,064)^2 \cdot 32,0133}_{0.1311} + \underbrace{\frac{(0,15)^2}{3}}_{0.0075}$$

$$\approx 0,1386\ (mA)^2$$

3) $$\sigma_n^2 = \underbrace{\left(\frac{\partial n}{\partial i}\right)^2 \sigma_i^2}_{K_2^2 \cdot \sigma_i^2} + \frac{(h_3)^2}{3} = \underbrace{(12,80)^2 \cdot 0,1386}_{22,7082} + \underbrace{\frac{(0.5)^2}{3}}_{0,0833}$$

$$\approx 22,7915$$

4) $$\sigma^2_{v_M} = \left(\frac{\partial v_M}{\partial n}\right)^2 \sigma_n^2 + \frac{(h_4)^2}{3} = \left(K_3 \frac{1}{2}(n-51)^{-\frac{1}{2}}\right)^2 \sigma_n^2 + \frac{(h_4)^2}{3}$$

$$\frac{\partial v_M}{\partial n} = \frac{\partial}{\partial n}\left\{K_3 (n-51)^{1/2}\right\} = K_3 \frac{\partial}{\partial n}\left\{(n-51)^{1/2}\right\}$$

$$= K_3 \frac{1}{2}(n-51)^{-\frac{1}{2}}$$

$$\approx \frac{K_3^2 \cdot \frac{1}{4}}{(n-51)} \cdot \sigma_n^2 + \frac{(h_4)^2}{3} = \frac{(1.43)^2 \cdot \frac{1}{4}}{(148-51)} \cdot 22,7915 + 0$$

$$\approx 0,1201\ (m/s)^2$$

$$\Rightarrow \sigma_e = \sigma_{v_M} = \sqrt{0,1201} \approx \underline{\underline{0,35\ m/s}}$$

$$\left.\begin{array}{l} e = v_M - v_T \\ \text{and } \sigma_{v_T} = 0 \end{array}\right\} \Rightarrow \sigma_e = \sigma_{v_M}$$

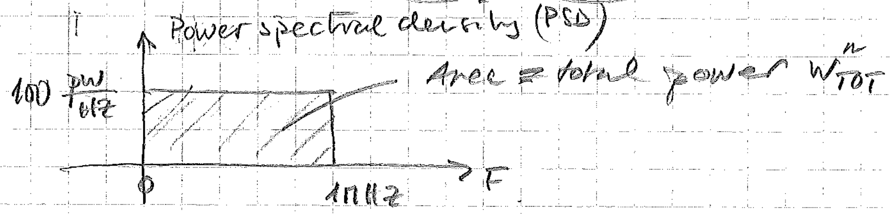
(11) a) $SNR = \frac{S_{rms}}{n_{rms}}$ rms = root mean square

~~sinusoidal~~

Signal : $A \cdot \sin(2\pi F t)$; $A = 1.4\ mV$
; $F = 5\ kHz$

$\rightarrow S_{rms} = \frac{A}{\sqrt{2}} = \frac{1.4 \cdot 10^{-3}}{\sqrt{2}}\ V.$

Noise :

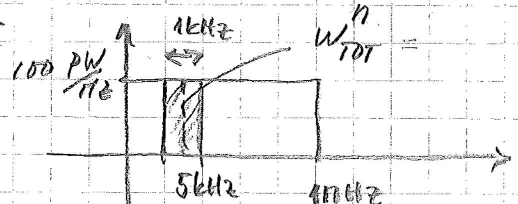


Total power ; $W^n_{TOT} = 100 \cdot 10^{-12}\ W/Hz \cdot 10^6\ Hz = 10^{-4}\ W$

$n^2_{rms} = W^n_{TOT} \rightarrow n_{rms} = 10^{-2}\ V$

$SNR_{before} = 20\ {}^{10}\log\left(\frac{\frac{1.4}{\sqrt{2}} \cdot 10^{-3}}{10^{-2}}\right) \approx -20{,}1\ dB$

Filtering.



$W^n_{TOT} = 100 \cdot 10^{-12} \cdot 10^3 = 10^{-7}\ W$

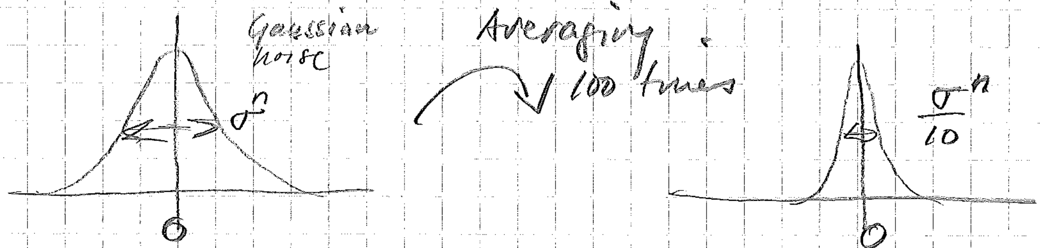
$n^2_{rms} = W^n_{TOT} \Rightarrow n_{rms} = \sqrt{W^n_{TOT}} = \sqrt{10^{-7}} = \sqrt{10 \cdot 10^{-8}} = \sqrt{10} \cdot 10^{-4}\ V$

$SNR_{after} = 20\ {}^{10}\log\left(\frac{\frac{1.4}{\sqrt{2}} \cdot 10^{-3}}{\sqrt{10} \cdot 10^{-4}}\right) = +9{,}9\ dB$

(11) cont..

b)

Averaging 100 times $\rightarrow$ $\sigma_{AV}^{n} = \frac{\sigma^{n}}{\sqrt{100}} = \frac{\sigma^{n}}{10}$



$n_{rms}^{2} = \sigma^{2}$ when zero-mean process.

$\rightarrow \quad n_{rms}^{AV} = \frac{n_{rms}}{10} = \frac{\sqrt{10}\cdot 10^{-4}}{10} = \sqrt{10}\cdot 10^{-5}\ V.$

$SNR = 20\,{}^{10}\log\left(\frac{\frac{1.4}{\sqrt{2}}\cdot 10^{-3}}{\sqrt{10}\cdot 10^{-5}}\right) = \underline{\underline{+29{,}9\ dB}}$

c)

+30 dB

Filtering: $-20{,}1$ dB $\rightarrow$ $+9.9$ dB

Averaging $+9.9$ dB $\rightarrow$ $+29.9$ dB

+20 dB

BP-filtering is more "powerful" compare to averaging