Home Assignment 1 - Multivariable calculus, 2010.

Deadline: 23.09.2010.

1. Calculate the following limit (or show that it does not exist)

$$\lim_{(x,y)\to(2,-2)} \frac{\ln(1+x^2-y^2)}{x^4-y^4}. \qquad \textit{(1/4 p)}$$

2. We are given the function

$$f(x,y) = \begin{cases} \frac{x^2 y}{x^2+y^2}\,, & (x,y) \neq (0,0) \\ 0\,, & (x,y) = (0,0) \end{cases}$$

   Show that $f_x(0,0)$ and $f_y(0,0)$ both exist.
   Also show that f_x and f_y are *not* continuous at $(0,0)$. *(1/4 p)*

3. A wall makes an angle of 120° with the ground. A ladder of length $10\,(m)$ is leaning against the wall and its top is sliding down the wall at a (constant) rate of $3\,(m/s)$. How fast is the area of the triangle formed by the ladder, the wall, and the ground changing at a time when the ladder makes an angle of 45° with the ground ? *(1/2 p)*

4. The equation $e^{x+y+z} + z - x - y - xy = 1$ implicitly defines a function $z = f(x,y)$ for which $f(0,0) = 0$. Show that $(0,0)$ is a critical point. Also determine whether this point is a (local) extremum. *(1/2 p)*

5. A company produces two different products. The production volumes are x and y, respectively. The running production costs are given by the function $c(x,y) = x^2 + 2xy + 2y^2$, while the retail prices (per production unit) are $p_x(x) = 10 - x$ and $p_y(y) = 20 - y$.
   How should the company adjust production volumes in order to maximize the profit ? *(1/2 p)*