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Akademin för informationsteknologi

Methodologies for Approximation of Unary Functions and Their Implementation in Hardware

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Akademisk avhandling som för avläggande av doktorsexamen vid Högskolan i Halmstad försvaras offentligt fredagen den 2 september 2016, kl. 13.00 i Wigforssalen, Visionen, Högskolan i Halmstad.

Avhandlingen försvaras på engelska.

Opponent: Prof. Michael Faulkner, Victoria University, Melbourne, Australien.
Betygsnämnd: Docent Oscar Gustafsson, Linköpings universitet och
Dr. Hans Hellsten, Saab AB; Prof. Warwick Tucker, Uppsala universitet.
Ordförande vid disputationen: Prof. Walid Taha.

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Abstract

Applications in computer graphics, digital signal processing, communication systems, robotics, astrophysics, fluid physics and many other areas have evolved to become very computation intensive. Algorithms are becoming increasingly complex and require higher accuracy in the computations. In addition, software solutions for these applications are in many cases not sufficient in terms of performance. A hardware implementation is therefore needed. A bottleneck in the algorithms is the performance of the approximations of unary functions, such as trigonometric functions, logarithms and the square root, as well as binary functions such as division. The challenge is therefore to develop a methodology for the implementation of approximations of unary functions in hardware that can cope with the growing requirements. The requirements are fast execution time, low complexity using basic hardware operations, and – since many applications are battery powered – low power consumption. To ensure appropriate performance of the entire computation in which the approximation is a part, the characteristics and distribution of the approximation error are also things that must be possible to manage.

The new approximation methodologies presented in this thesis are of the type that aims to reduce the sizes of the look-up tables by the use of auxiliary functions. They are founded on a synthesis of parabolic functions by multiplication – instead of addition. Three approximation methodologies have been developed; the two last being further developments of the first.

For some functions, such as roots, inverse and inverse roots, a straightforward solution with an approximation is not manageable. Since these functions are frequent in many computation intensive algorithms, it is necessary to find very efficient implementations of these functions. New methods for this are also presented in this thesis. They are all founded on working in a floating-point format, and, for the roots functions, a change of number base is also used.

Tools for error analysis have been developed as well. The characteristics and distribution of the approximation error in the new methodologies are presented and compared with existing state-of-the-art methods. The verification and evaluation of the solutions have to a large extent been made as comparative ASIC implementations with other approximation methods, separately or embedded in algorithms. In summary, the new approximation methodologies presented are found to well meet the demanding requirements that exist in this area.