Exam for Random Processes, 7.5 ECTS

April 28, 2011, 9.00 – 13.00

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
Allowed aids: Summary of formulae attached to the exam, calculator and Math. Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://dixon.hh.se/erja` → `Teaching` → `Random processes` → `Previous exams`

1. Prove that if $\{X_t\}$ is a weakly stationary random process in continuous time with covariance function $r_X(\tau)$, then the covariance function, $r_{X'}(\tau)$, of the derivative process, $\{X'_t\}$, is $-r''_X(\tau)$. (5p)

2. Let $\{A_t\}$ be an $MA(2)$ process with $c_0 = c_2 = \sigma_\epsilon^2 = 1$ and $c_1 = 0$.
   (a) Determine the spectral density function $R_A(f)$ of $\{A_t\}$. (4p)
   (b) What kind of process is $\{B_t\}$ if $B_t = A_t + A_{t-1}$ for all t?
   Derive the covariance function $r_B(\tau)$ of $\{B_t\}$. (4p)

3. The *Gauss-Markov process* $\{Y_t : t \in \mathbb{R}^+\}$ is defined by $Y_t = e^{-\alpha t} W(e^{2\alpha t})$ for all $t \in \mathbb{R}^+$ where $\alpha > 0$ and $\{W(t)\}$ is a Wienerprocess with $V(W(1)) = \sigma^2$.
   (a) Prove that the process $\{Y_t\}$ is strongly stationary. (5p)
   (b) Let $\alpha = 0.1$ and $\sigma^2 = 1$ and calculate $V(\int_0^\infty Y_t\,dt)$. (4p)

4. Assume that $\{N_t : t \in \mathbb{R}^+\}$ is Poisson process with intensity $\lambda = 3$. Calculate
   (a) $C(N_7, N_{11})$. (2p)
   (b) $E(Y_t)$ where $Y_t \in Exp(N_t + 1)$. (6p)

GOOD LUCK!