Exam for Stochastic models in Discrete time
3.75 ECTS

Master's program of Financial Mathematics
April 1, 2011, 9.00 – 13.00

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
ECTS bounds: 12p ⇒ grade E, 15p ⇒ grade D, 18p ⇒ grade C, 21p ⇒ grade B, 24p ⇒ grade A.
Allowed aids: Summary of formulae attached to the exam, calculator and dictionary.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet.
The proper solutions will be available on the internet at `http://www.hh.se/staff/erja` →
→ `Teaching` → `Financial Mathematics` → `Stochastic models` → `Previous exams`

1. Let $X = \{(X_n, \mathcal{F}_n) : n \geq 0\}$ be a stochastic sequence with $E(|X_n|) < \infty$. Show that if X is a generalized martingale, then X is a martingale transformation. (5p)

2. Let $\{X_t : t \in \mathbb{Z}\}$ be an $AR(1)$ process with parameters $a_0 = 100$ and $a_1 = 0.9$. Determine the
   (a) conditional probability $P(X_n \leq 111 \,|\, X_{n-1} = 11)$ if $\sigma_\epsilon^2 = 0.81$. (4p)
   (b) noise variance σ_ϵ^2 such that the process $\{X_n\}$ is stationary. (5p)

3. Assume $\{h_n\}$ is distributed according to the $HARCH(p)$ model with $a_0 = 1$. Show that
$$\sum_{k=1}^{p} k a_k \;>\; 0$$
(6p)

4. Consider the random walk $\{X_n : n = 0, 1, 2, \ldots\}$.
   (a) Calculate $V(X_3)$. (4p)
   (b) Prove that the geometric random walk $\{Y_n : n = 0, 1, 2, \ldots\}$, where $Y_n = e^{X_n}$, is a submartingale with respect to the filtration $\{\mathcal{F}_n\}$ where $\mathcal{F}_n = \sigma(X_1, X_2, \ldots, X_n)$. (6p)

GOOD LUCK!