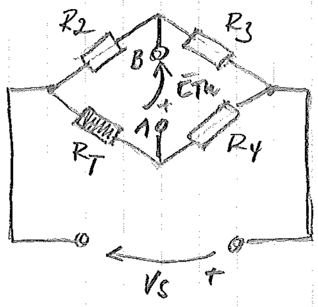
(9)

a)




$\times\ R_T = R_0(1+\alpha T)$

$\begin{cases} R_0 = 100\,\Omega \\ \alpha = 4\cdot 10^{-3}\ 1/°C \end{cases}$

$\times\ V_s = 5\,V$

Input: $T \in [0, 100]\ °C$

Output: $E_{Th} \in [0, 20]\ mV$

Balanced output: $E_{Th} = 0V$ when $R_T = R_{min} = R_0$

$$E_{Th} = V_s\left(\frac{R_0}{R_0+R_4} - \frac{R_2}{R_2+R_3}\right)$$

$$\frac{E_{Th}}{V_s} = \left(\frac{1}{1+R_4/R_0} - \frac{1}{1+R_3/R_2}\right)$$

$$E_{Th} = 0 \Rightarrow \frac{R_4}{R_0} = \frac{R_3}{R_2} \Rightarrow \boxed{\begin{matrix} R_2 = R_0 = 100\,\Omega \\ R_4 = R_3 \end{matrix}}$$

Unbalanced output: $E_{Th} = 20\,mV$ when $T = 100\,°C$

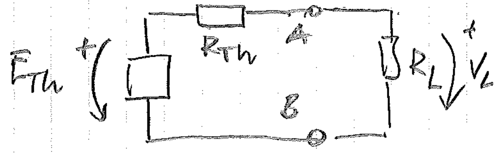
$$E_{Th} = V_s\frac{R_2}{R_3}\alpha T \Rightarrow 20\cdot 10^{-3} = 5\frac{R_2}{R_3}4\cdot 10^{-3}\cdot 100$$

$$\Rightarrow \frac{R_2}{R_3} = \frac{1}{100} \quad \text{and } R_3 = 100\,R_2 \ (R_2 = 100\,\Omega) = 10\,k\Omega$$

Sum up: $R_2 = 100\,\Omega$

$R_3 = R_4 = 10\,k\Omega$

b) Make a Thevenin circuit




$R_L = 5\,k\Omega$

$R_{Th} = R_T // R_4 + R_2 // R_3$

$V_L = E_{Th}\cdot\frac{R_{Th}}{R_{Th}+R_L}$

$T = 0°C \rightarrow \left.\begin{matrix} R_{Th} = 100 // R_4 + R_2 // R_3 \simeq 200\,\Omega \\ E_{Th} = 0V \end{matrix}\right\} \Rightarrow V_L = 0V$

$T = 100°C \Rightarrow \left.\begin{matrix} R_{Th} = 140 // R_4 + R_2 // R_3 \simeq 240\,\Omega \\ E_{Th} = 20\,mV \end{matrix}\right\} \Rightarrow V_L = 19{,}1\,mV$

So $V_L \in [0, 19.1]\ mV$

(10)

$$F(s) \longrightarrow \boxed{K} \longrightarrow \boxed{G_1(s)} \longrightarrow \boxed{G_2(s)} \longrightarrow F_{rd}(s)$$

$$\begin{cases} K = 20 \\ G_1(s) = 10^{-3} \cdot \dfrac{\tau \cdot s}{1+\tau \cdot s} & ; \ \tau = 0.1\,s \\ G_2(s) = \dfrac{50}{\left(\frac{1}{\omega_n^2} s^2 + \frac{2\xi}{\omega_n} \cdot s + 1\right)} & ; \ \begin{array}{l} f_n = 40\ kHz \\ \xi = 0.01 \end{array} \end{cases}$$

$$F(t) = 50\left(\sin(400 \cdot t) + \tfrac{1}{3}\sin(1200 \cdot t) + \tfrac{1}{5}\sin(2000 \cdot t)\right)\ N$$

$\omega = 400\ rad/s$ $\qquad$ $\omega = 1200\ rad/s$ $\qquad$ $\omega = 2000\ rad/s$

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$$G(\omega) = G(s)\big|_{s=j\omega}$$

$\times$ $$G_1(\omega) = 10^{-3} \frac{0.1\,j\omega}{1 + 0.1\,j\omega}$$

for $\omega \geq 400\ rad/s$ $\qquad$ $1 + 0{,}1\,j\omega \simeq 0{,}1\,j\omega$

$$\Rightarrow G_1(\omega) \simeq 10^{-3}$$

$\times$ $$G_2(\omega) = \frac{50}{-\left(\frac{\omega}{\omega_n}\right)^2 + j\,2\xi\left(\frac{\omega}{\omega_n}\right) + 1}$$

for $\omega \in [400, 2000]$ $\qquad$ $-\left(\frac{\omega}{\omega_n}\right)^2 + j\,2\xi\left(\frac{\omega}{\omega_n}\right) + 1 \simeq 1$

$\omega \ll \omega_n$

$$\Rightarrow G_2(\omega) \simeq 50$$

Sum up: $\quad G(\omega) \approx \underbrace{K}_{=20} \cdot 10^{-3} \cdot 50 = 1 \qquad \omega \in [400, 2000]$

And: $\quad |G(\omega)| \simeq 1$

$\quad \quad \ \ Arg\{G(\omega)\} \simeq 0°$ $\qquad$ when $\omega \in [400, 2000]\ rad/s$

(11)

a) $e=2{,}3\cdot10^{-4} \longrightarrow \boxed{1)} \longrightarrow E=\frac{1}{2}V_s\cdot G\cdot e \quad [V]$

$$\begin{cases}\bar{V}_s = 10V \ ; \ \sigma_{V_s} = 10mV \\ \bar{G} = 2{,}10 \ ; \ \sigma_G = 0{,}02\end{cases}$$

× $\bar{E} = \frac{1}{2}\bar{V}_s\cdot\bar{G}\cdot e = \frac{1}{2}\cdot 10\cdot 2{,}10\cdot 2{,}3\cdot 10^{-4} = 2{,}415\ mV$

× $\sigma_E^2 = \underbrace{\left(\frac{\partial E}{\partial V_s}\right)}_{\frac{1}{2}\bar{G}\cdot e}^2\sigma_{V_s}^2 + \underbrace{\left(\frac{\partial E}{\partial G}\right)}_{\frac{1}{2}V_s\cdot e}^2\cdot\sigma_G^2 = 6{,}34\cdot10^{-10}\ V^2 = 6{,}34\cdot10^{-4}\ (mV)^2$

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$E \longrightarrow \boxed{2)} \longrightarrow i = a + K\cdot E + K_{II}E\cdot\Delta T_a + K_I\Delta T_a \quad [mA]$

$$\begin{cases}\bar{a} = 4mA \ ; \ \sigma_a = 0{,}01\ mA \\ \bar{K} = 1{,}52\ mA/mV \ ; \ \sigma_K = 0 \\ \bar{K}_{II} = 0{,}005\ mA/mV\cdot{}^\circ C \ ; \ \sigma_{K_{II}} = 0 \\ \bar{K}_I = 0{,}01\ mA/{}^\circ C \ ; \ \sigma_{K_I} = 0 \\ \overline{\Delta T_a} = +10^\circ C \ ; \ \sigma_{\Delta T_a} = 6{,}2^\circ C\end{cases}$$

× $\bar{i} = \bar{a} + \bar{K}\cdot\bar{E} + \bar{K}_{II}\cdot\bar{E}\cdot\overline{\Delta T_a} + \bar{K}_I\cdot\overline{\Delta T_a} = 7{,}89\ mA$

× $\sigma_i^2 = \underbrace{\left(\frac{\partial i}{\partial a}\right)}_{1}^2\sigma_a^2 + \underbrace{\left(\frac{\partial i}{\partial \Delta T_a}\right)}_{(K_{II}\cdot E + K_I)}^2\sigma_{\Delta T_a}^2 + \underbrace{\left(\frac{\partial i}{\partial E}\right)}_{(K + K_{II}\cdot\Delta T_a)}^2\cdot\sigma_E^2 = 1{,}46\cdot10^{-2}\ (mA)^2$

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$i \longrightarrow \boxed{3)} \longrightarrow e_m = K\cdot i + a$

$$\begin{cases}\bar{K} = \frac{10^{-3}}{16}\ 1/mA \ ; \ \sigma_K = 0 \\ \bar{a} = -\frac{10^{-3}}{4} \ ; \ \sigma_a = 10^{-5}\end{cases}$$

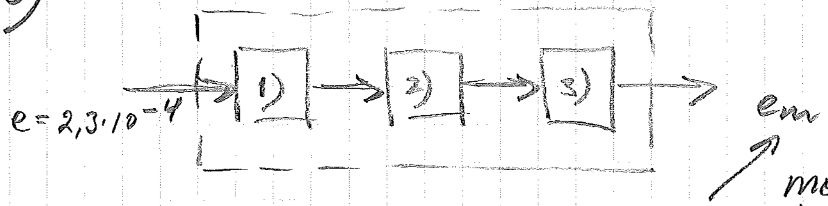
× $\bar{e}_m = \bar{K}\cdot\bar{i} + \bar{a} = 2{,}431\cdot10^{-4}$

× $\sigma_{e_m}^2 = \underbrace{\left(\frac{\partial e_m}{\partial a}\right)}_{1}^2\sigma_a^2 + \underbrace{\left(\frac{\partial e_m}{\partial i}\right)}_{K}^2\sigma_i^2 = 1{,}57\cdot10^{-10}$

$\Rightarrow \quad \bar{e}_m = 2{,}43\cdot10^{-4}$

$\sigma_{e_m} = 1{,}25\cdot10^{-5}$ //

(11) b)




measured over time
⇒ histogram

Calculate $m$ and $\sigma$ from the histogram.

$$m = \overline{e_m} = \frac{1}{N} \sum_{i=1}^{N} e_{m_i}$$

$$= \frac{1}{300}\left[7 \cdot 2{,}196 \cdot 10^{-4} + 10 \cdot 2{,}247 \cdot 10^{-4} + \dots + 7 \cdot 2{,}658 \cdot 10^{-4}\right]$$

$$\approx 2{,}43 \cdot 10^{-4}$$

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} \left(e_{m_i} - \overline{e_m}\right)^2$$

$$= \frac{1}{299}\left(7\left(2{,}196 \cdot 10^{-4} - 2{,}43 \cdot 10^{-4}\right)^2 + \dots + 7\left(2{,}658 \cdot 10^{-4} - 2{,}43 \cdot 10^{-4}\right)^2\right]$$

$$\approx 1{,}065 \cdot 10^{-10}$$

and $\sigma = 1{,}03 \cdot 10^{-5}$