Exam in Signal analysis and representation, 7.5 credits.
Course code: dt8010
Date: 2011-01-05

Allowed items on the exam:
Tables of Signal processing formulas.
Tables of Mathematical formulas.
Calculator.

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Maximum points: 8.
In order to pass the examination with a grade 3 a minimum of 3.3 points is required.
To get a grade 4 a minimum of 4.9 points is required, and to get a grade 5 a minimum of 6.5 points is required.

Give your answer in a readable way and motivate your assumptions.

Good Luck!

1. (2p)
A LTI system is represented by the difference equation:

$$y(n)-\frac{9}{4}y(n-1)+\frac{1}{2}y(n-2)=\frac{3}{4}x(n)-\frac{5}{8}x(n-1).$$

a) Sketch the pole-zero pattern of the system H(z). (0.8 p)
b) Specify the ROC of H(z) and determine h(n) for the following conditions:
i) The system is stable. (0.6p)
ii) The system is causal. (0.6p)

2. (2p)
A causal FIR-system is described by its impulse response:

$$h(n)=\frac{1}{3}\partial(n)+\frac{1}{3}\partial(n-1)+\frac{1}{3}\partial(n-2).$$

a) Compute the frequency response function H(ω), and sketch the magnitude- and phase-function for $0\leq \omega \leq 2\pi$.
Present H(ω) as $H(\omega)=H_{real}(\omega)e^{-j\omega(M-1)/2}$ where $H_{real}(\omega)$ is a real function and M is the length of h(n). (1p)
b) Determine the steady state response y(n) of the system to the input signal:

$$x(n)=2\sin\left(\frac{\pi}{3}(n-1)\right)+0.5\cos\left(\frac{2\pi}{3}n\right).$$ (1p)

3. (2p)
Consider the causal system

$$H(z)=\frac{1}{1-z^{-1}+\frac{1}{2}z^{-2}}$$

Determine:
a) The impulse response h(n). (1p)
b) The step response when the system is initially relaxed. (1p)

4. (2p)
a) Compute the 8-point DFT of the causal system:

$$h(n)=\partial(n-3)$$

Present H(k) as: $e^{-jkM}H_{real}(k)$. (0.6p)

b) Sketch the magnitude- and the phase-function of the system. (0.6p)
c) Compute the response to the input

$$x(n)=\left[0.8\cos(\frac{3\pi}{4}n)+1.2\right]u(n).$$ (0.8p)

TABLE 3.3 Some Common z-Transform Pairs

	Signal, $x(n)$	z-Transform, $X(z)$	ROC
1	$\delta(n)$	1	All z
2	$u(n)$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3	$a^n u(n)$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
4	$na^n u(n)$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
5	$-a^n u(-n-1)$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
6	$-na^n u(-n-1)$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
7	$(\cos \omega_0 n)u(n)$	$\dfrac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
8	$(\sin \omega_0 n)u(n)$	$\dfrac{z^{-1}\sin\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
9	$(a^n \cos \omega_0 n)u(n)$	$\dfrac{1-az^{-1}\cos\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
10	$(a^n \sin \omega_0 n)u(n)$	$\dfrac{az^{-1}\sin\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$