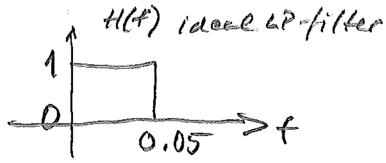
(9)

$$T(t) = 10\left(\sin(\omega_0 t) + \tfrac{1}{2}\sin(2\omega_0 t) + \tfrac{1}{3}\sin(3\omega_0 t) + \tfrac{1}{4}\sin(4\omega_0 t)\right)$$

period of $T(t)$ is $63\,s$ $\Rightarrow$ $f_0 = \frac{1}{63} \approx 0{,}016$ Hz

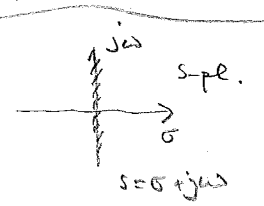


$f_0 = 0.016$

$2f_0 = 0{,}032$

$3f_0 = 0{,}048$

$\cancel{4f_0 = 0{,}064}$ is stopped by the ideal lp-filter



$$G(s) = \frac{1}{1+5\cdot s}$$

$$G(\omega) = G(s)\big|_{s=j\omega} = \frac{1}{1+j5\omega}$$

a)

$$T_m(t) = 10\Big[\,|G(\omega_0)|\cdot\sin(\omega_0 t + \mathrm{Arg}\{G(\omega_0)\}) + \tfrac{1}{2}|G(2\omega_0)|\sin(2\omega_0 t + \mathrm{Arg}\{G(2\omega_0)\}) + \tfrac{1}{3}|G(3\omega_0)|\sin(3\omega_0 t + \mathrm{Arg}\{G(3\omega_0)\})\Big]$$

$$\begin{cases} |G(\omega)| = \dfrac{1}{\sqrt{1^2 + (5\omega)^2}} \\[2ex] \mathrm{Arg}\, G(\omega) = -\mathrm{Arg}\{1+j5\omega\} \end{cases}$$

ω	$\lvert G(\omega)\rvert$	$\mathrm{Arg}\, G(\omega)$	
ω_0	0,895	$-26{,}5°$	
$2\omega_0$	0,708	$-44{,}9°$	
$3\omega_0$	0,556	$-56{,}2°$	

$\omega_0 = 2\pi f_0 = \frac{2\pi}{63} \approx 0{,}1$ rad/s

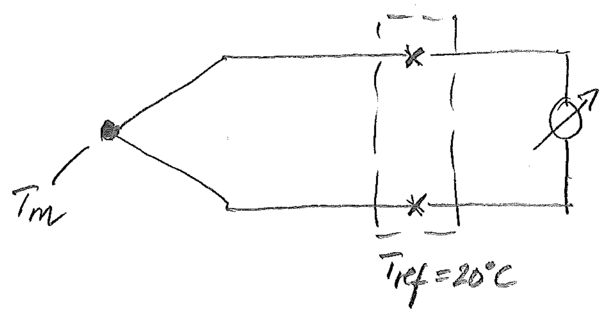
$$T_m(t) = 10\Big[0{,}895\sin(\omega_0 t - 26{,}5°) + \frac{0.708}{2}\sin(2\omega_0 t - 44{,}9°) + \frac{0{,}556}{3}\sin(3\omega_0 t - 56{,}2°)\Big]$$

(9) cont.

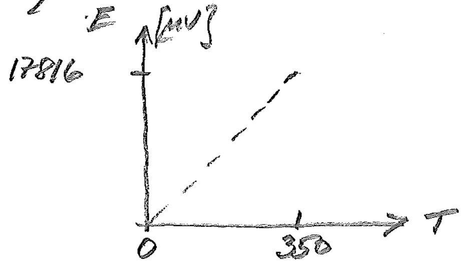
b)
$$E(t) = T_m(t) - T(t)$$
$$= 10\cdot[0{,}895\sin(\omega_0 t - 26{,}5^\circ) - \sin(\omega_0 t)]$$
$$+ \frac{10}{2}[0{,}708\sin(2\omega_0 t - 44{,}9^\circ) - \sin(2\omega_0 t)]$$
$$+ \frac{10}{3}[0{,}556\sin(3\omega_0 t - 56{,}2^\circ) - \sin(3\omega_0 t)]$$
$$- \frac{10}{4}\sin(4\omega_0 t)$$

---

(10)



a) Full scale $T \in [0, 350]$ °C see table!



$$E^{line} = k\cdot T \;;\quad k = \frac{17816}{350} \simeq 50{,}9\ \mu V/°C$$

$$N(100) = E^{table}_{100} - E^{line}(100) = 4277 - \frac{17816}{350}\cdot 100 = -813\ \mu V$$

$$N(300) = E^{table}_{300} - E^{line}(300) = 14860 - \frac{17816}{350}\cdot 300 = -411\ \mu V$$

% of full scale

$$N(100) = -\frac{813}{17816}\cdot 100 \simeq \underline{\underline{-4{,}6\%}}$$

$$N(300) = -\frac{411}{17816}\cdot 100 \simeq \underline{\underline{-2{,}3\%}}$$

(10) cont.

b) $\boxed{E_{T,0} = a_1 \cdot T + a_2 \cdot T^2}$; $T \in [100, 200]$ °C

Two unknowns $\rightarrow$ Two points (T, E) are used.

(several points at LS-method can also be used).

Points (T, E):

$(100, 4277)$ and $(200, 9286)$ from table

$\Rightarrow$ two equations:

$$\begin{cases} 4277 = a_1 100 + a_2 \cdot 100^2 & \text{point } (100, 4277) \\ 9286 = a_1 \cdot 200 + a_2 \cdot 200^2 & \text{point } (200, 9286) \end{cases}$$

$$\Rightarrow \begin{cases} a_1 = 39{,}1 \ \mu V/°C \\ a_2 = 36{,}6 \cdot 10^{-3} \ \mu V/°C^2 \end{cases}$$

---

c) $\boxed{E_{T,0} = E_{T,20} + E_{20,0}}$; $E_{T,20} = 5761 \ \mu V$

$E_{20,0} = 789 \ \mu V$ from table

$\Rightarrow E_{T,0} = 5761 + 789 = 6550 \ \mu V$

$\boxed{E_{T,0} = a_1 T + a_2 \cdot T^2} \Rightarrow$

$6550 = a_1 \cdot T + a_2 \cdot T^2$ find T

$a_2 T^2 + a_1 T - 6550 = 0$ $T = 147.2$

$$T^2 + \frac{a_1}{a_2} T - \frac{6550}{a_2} = 0$$

$$T = -\frac{a_1}{2 \cdot a_2} \pm \sqrt{\left(\frac{a_1}{2 \cdot a_2}\right)^2 + \frac{6550}{a_2}}$$

$$\simeq -534{,}2 \pm 681{,}4$$

$T = 147.2$ °C (Check from table $6550 \ \mu V \Rightarrow \sim 147$ °C)

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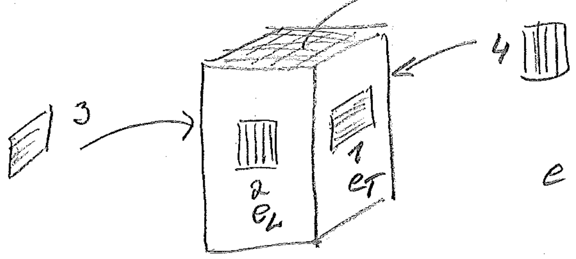
(11)

$A = 10 \times 10\ cm^2 = 100\ cm^2 = 100 \cdot 10^{-4}\ m^2 = 10^{-2}\ m^2$



$e = \frac{\Delta l}{l} \Rightarrow e_L < 0$ and $e_T > 0$

$(\Delta l < 0)$ "compressed" $\quad (\Delta l > 0)$ "tensiled"

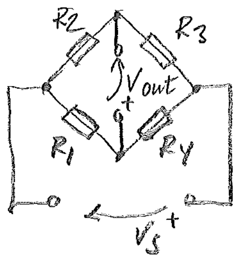
a) Strain gauges: $\boxed{\Delta R = R_0 \cdot G \cdot e}$ Basic formula!

$$\begin{cases} R_1 = R_3 = R_0 + R_0 \cdot G \cdot e_T = R_0(1 + G \cdot e_T) \ ; \ e_T > 0 \\ R_2 = R_4 = R_0 + R_0 \cdot G \cdot e_L = R_0(1 + G \cdot e_L) \ ; \ e_L < 0 \\ e_L = -\frac{F}{A \cdot E} = -\frac{10^5}{10^{-2} \cdot 2{,}1 \cdot 10^{11}} = -4{,}76 \cdot 10^{-5} \\ e_T = -\nu \cdot e_L = 0{,}29 \cdot 4{,}76 \cdot 10^{-5} \approx 1{,}38 \cdot 10^{-5} \end{cases}$$

$$\Rightarrow R_1 = R_3 = R_0(1 + G \cdot e_T) = 100(1 + 2.1 \cdot 1{,}38 \cdot 10^{-5}) \approx 100{,}0029\ \Omega$$

$$R_2 = R_4 = R_0(1 + G \cdot e_L) = 100(1 + 2.1(-4{,}76 \cdot 10^{-5})) \approx 99{,}99\ \Omega$$

b)

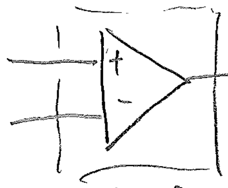


Temperature compensated:
T changes $\rightarrow \Delta R_1 = \Delta R_2 = \Delta R_3 = \Delta R_4$
and $\Delta V_{out} = 0$

High sensitivity:
$\left.\begin{array}{l} R_1 \text{ and } R_3 \text{ increase} \\ R_2 \text{ and } R_4 \text{ decrease} \end{array}\right\} \rightarrow \Delta V_{out}$ max

$$V_{out} = V_s \left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right) = V_s \left(\frac{100{,}0029 - 99{,}99}{199{,}9929} \right)$$

$$\approx V_s \cdot 6{,}45 \cdot 10^{-5}$$ small changes!



$V_{ampl} = A_{DM} \cdot V_{out}$

Diff. ampl.

(11) b.
cont.

Find minimum A_{DM} $\Leftrightarrow$ when V_{out} is max

V_{out} is max when V_S is max !

Maximum V_S is limited by the
maximum gauge current $i_{max} = 30\,mA$

$$i = \frac{V_S}{2R_0} \quad \text{(see figure of defl. bridge)}$$

$$\text{So } V_S \leq i_{max} \cdot 2R_0 = 30 \cdot 10^{-3} \cdot 2 \cdot 100 = 6V$$

$$V_S = 6V \Rightarrow V_{out} = 6 \cdot 6{,}45 \cdot 10^{-5} \approx 3{,}87 \cdot 10^{-4}\,V$$

$$V_{ampl} = A_{DM} \cdot V_{out} \quad ; \quad V_{ampl} = 1V$$

$$\Rightarrow 1 = A_{DM} \cdot 3.87 \cdot 10^{-4}$$

$$A_{DM} = \frac{1}{3{,}87 \cdot 10^{-4}} \approx 2580 \text{ ggr.}$$