Exam for Stochastic models in Discrete time
3.75 ECTS

Master's program of Financial Mathematics
October 30, 2010, 9.00 – 13.00

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
ECTS bounds: 12p ⇒ grade E, 15p ⇒ grade D, 18p ⇒ grade C, 21p ⇒ grade B, 24p ⇒ grade A.
Allowed aids: Summary of formulae attached to the exam, calculator and dictionary.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet.
The proper solutions will be available on the internet at `http://www.hh.se/staff/erja` → → `Teaching` → `Financial Mathematics` → `Stochastic models` → `Previous exams`

1. Prove the sufficiency part of the *Martingale criterion for absence of arbitrage.* (5p)

2. Let $\{X_n : n \in \mathbb{Z}\}$ be an $MA(\infty)$ process with parameters $\mu = 0$ and $b_k = 2^{-k}$ for $k = 0, 1, 2, 3, \ldots$
   (a) Determine the variance of X_n, $D(X_n)$. (4p)
   (b) Calculate $P(X_{n+1} \leq 1 \,|\, X_n = 1)$. (3p)
   (c) Assuming $b_0 = 1$, $b_1 = \frac{1}{2}$ and $b_k = 0$ for $k = 2, 3, 4, \ldots$ Prove that the process $\{X_n\}$ is mesokurtic. (5p)

3. Calculate the fourth moment $E(h_n^4)$ of an $ARCH(1)$ process $\{h_n\}$. (4p)

4. Let $\{h_n : n \in \mathbb{Z}^+\}$ be an asymetric random walk such that $h_0 = 0$ and $h_n = \sum_{k=1}^{n} U_k$ for $n = 1, 2, 3, \ldots$ where the sequence $\{U_k\}$ consists of independent random variables such that $P(U_k = 1) = 1 - P(U_k = C) = p$ where $0 < p < 1$.
   (a) Determine the number C such that $\{h_n\}$ is a martingale with respect to the flow $\{\mathcal{F}_n\}$ where $\mathcal{F}_n = \sigma(U_1, U_2, \ldots, U_n)$. (4p)
   (b) Assume that $C = -1$ and that the observations $h_1 = 1$, $h_2 = 2$, $h_3 = 1$, $h_4 = 2$, $h_5 = 3$ are made. Calclulate the maximum likelihood estimator of p. (5p)

GOOD LUCK!