4 Diskret spektrum (diskret tid)

$$r(\tau) = \sum_k A_k \cos(2\pi f_k t + \phi_k) \quad \text{där } 0 \le f_k \le \frac{1}{2}$$

enl. Eulers formel

samband mellan vvf, kvf, spektr.

Sats 6 Vikningseffekten (s. 75)

Om $\{Y_t : t \in \mathbb{R}\}$ svagt stationär

observeras i tidpunkterna

$t = \dots, -2d, -d, 0, d, 2d, \dots$

$Z_t = Y_t$ då $t = 0, \pm d, \pm 2d, \dots$

så $m_Z = m_Y$ och $r_Z = r_Y$

där $r_Z(\tau) = \displaystyle\int_{-\frac{1}{2d}}^{\frac{1}{2d}} e^{i2\pi f\tau} R_Z(f)\,df$ $\quad \tau = 0, \pm d, \pm 2d, \dots$

$$R_Z(f) = \sum_{k=-\infty}^{\infty} R_Y\left(f + \frac{k}{d}\right) \quad -\frac{1}{2d} < f \le \frac{1}{2d}$$

Bevis av sats 6:

Antag $\{Y_t\}$ svagt stationär och att

$Z_t := Y_t$ för $t \in \{nd : n \in \mathbb{Z}\}$.

Då är

$$r_Z(\tau) = r_Y(\tau) = \int_{-\infty}^{\infty} e^{i2\pi f\tau} R_Y(f)\,df =$$

$$\overset{\{\tau = nd\}}{=} \int_{-\infty}^{\infty} e^{i2\pi f nd} R_Y(f)\,df$$

$\{$Dela upp $(-\infty,\infty)$ $\ldots, \left(-\frac{3}{2d}, -\frac{1}{2d}\right], \left(-\frac{1}{2d}, \frac{1}{2d}\right], \left(\frac{1}{2d}, \frac{3}{2d}\right], \ldots$

$$= \sum_{k=-\infty}^{\infty} \int_{\frac{2k-1}{2d}}^{\frac{2k+1}{2d}} e^{i2\pi f nd} R_Y(f)\,df$$

$$\left\{ \begin{array}{lll} \text{Substitution} & f = (2k-1)/(2d) & f = (2k+1)/(2d) \\ f = v + k/d & v = f - k/d & v = f - k/d \\ df = dv & \quad = \frac{2k-1}{2d} - \frac{k}{d} & \quad = \frac{2k+1-2k}{2d} \\ & \quad = \frac{2k-1-2k}{2d} = -\frac{1}{2d} & \quad = \frac{1}{2d} \end{array} \right\}$$

$$= \sum_{k=-\infty}^{\infty} \int_{-1/2d}^{1/2d} e^{i2\pi(v+k/d)nd} R_Y(v+k/d)\,dv$$

$$= \sum_{k=-\infty}^{\infty} \int_{-1/2d}^{1/2d} e^{i2\pi vnd} \underbrace{e^{i2\pi \underbrace{kn}_{\text{heltal}}}}_{=1} R_Y(v+k/d)\,dv$$

$$= \int_{-1/2d}^{1/2d} \sum_{k=-\infty}^{\infty} e^{i2\pi vnd} R_Y(v+k/d)\,dv$$

$$= \int_{-1/2d}^{1/2d} e^{i2\pi v\underbrace{nd}_{\tau}} R_Z(v)\,dv \quad \text{där } R_Z(v) = \sum_{k=-\infty}^{\infty} R_Y\left(v+\frac{k}{d}\right)$$

$\square$