Exam for Random Processes, 7.5 ECTS

December 20, 2010, 9.00 – 13.00

Max number of points: 30.
Halmstad University grading bounds: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
Allowed aids: Summary of formulae attached to the exam, calculator and Math. Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53, 0702-822 844).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://dixon.hh.se/erja` → `Teaching` → `Random processes` → `Previous exams`

1. Prove that if $\boldsymbol{X} = (X_1, X_2, \ldots, X_n)$ is distributed according to the multivariate normal distribution and $C(X_i, X_j) = 0$ for all $i \neq j$, then X_i is independent of X_j for all $i \neq j$. (4p)

2. Let $\{X_t : t \in \mathbb{Z}\}$ be an $MA(2)$ process with $c_0 = 2$, $c_1 = 0$, $c_2 = 1$ and $\sigma_\epsilon^2 = \frac{1}{2}$. Calculate $E(X_t^2)$. (3p)

3. Consider the process $\{Y_t : t \in \mathbb{R}^+\}$ where $Y_t = \frac{3}{\sqrt{t}} N_t - 6\sqrt{t}$ for all t and $\{N_t\}$ is a Poisson process with intensity $\lambda = 2$. Calculate
   (a) $E(Y_t)$ and $V(Y_t)$. (2p)
   (b) approximately $P(Y_{100} \leq 180)$. (3p)
   (c) Is $\{Y_t\}$ strongly stationary? (3p)

4. Assume that a weakly stationary process $\{X_t : t \in \mathbb{R}\}$ with ef $m_X = 2$ is filtered with the impulse response $h(t) = \delta_0(t) + 0.5\delta_{0.5}(t)$. Determine the expectation function of the output signal. (2p).

5. Let $\{X_t\}$ be a weakly stationary Gaussian process in continuous time with ef $m_X = 2$ and cvf $r_X(\tau) = \frac{\pi^2}{\pi^2 + \tau^2}$. Determine the
   (a) spectral density function of $\{X_t\}$. (3p)
   (b) cvf of the derivative process $\{X_t'\}$. (3p)

   Assume $\{X_t\}$ is sampled at time-points $t = 0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \ldots$
   (c) What is the spectral density function of the sampled process? (4p)

6. Suppose that $\{X_t : t \in \mathbb{Z}\}$ is a weakly stationary process with cvf $r_X(\tau) = (|\tau|!)^{-1}$. A sample of size 100 of this process is about to be observed and from this sample the average value will be calculated. What is approximately the variance of this average value? (3p)

   (*Hint:* $\sum_{k=0}^{\infty} \frac{1}{k!} = e$.)

GOOD LUCK!