

Home Assignment 3 - Multivariable calculus, 2009.

Deadline: 23.10.2009.

1. Calculate $\iint_A (x^4 - y^4) e^{xy} dx dy$, where A is the region in the first quadrant bounded by the curves $xy = 1$, $xy = 2$, $x^2 - y^2 = 1$, and $x^2 - y^2 = 4$. (1/4 p)
2. Find the area of the region common to the interiors of the cardioids $r = 1 + \cos \theta$ and $r = 1 - \cos \theta$. (1/4 p)
3. Find the center of mass of a homogeneous thin plate (surface density, $\sigma = \text{const.}$) which is bounded by $x = e^2$, $y = 0$, and $y = \ln x$. (1/4 p)
4. Calculate $\iint_D \frac{1}{(x^2 + 4y^2)^2} dx dy$,
where $D = \{(x, y) \in \mathbb{R}^2 \mid x > 0, xy > 1\}$. (1/2 p)
5. Calculate $\iiint_K \frac{xyz^2}{1 + x^2 + 2y^2 + 3z^2} dx dy dz$,
where $K = \{(x, y, z) \in \mathbb{R}^3 \mid 1 \leq x^2 + 2y^2 + 3z^2 \leq 3, x \geq 0, y \geq 0\}$. (1/2 p)
6. Calculate the area of the part of the sphere $x^2 + y^2 + z^2 = 9$
for which $-1 \leq z \leq 2$. (1/4 p)