Exam in Signal analysis and representation, 7.5 credits.
Course code: dt8010
Date: 2009-10-26

Allowed items on the exam:
Tables of Signal processing formulas.
Tables of Mathematical formulas.
Calculator.

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Maximum points: 8.
In order to pass the examination with a grade 3 a minimum of 3.3 points is required.
To get a grade 4 a minimum of 4.9 points is required, and to get a grade 5 a minimum of 6.5 points is required.

Give your answer in a readable way and motivate your assumptions.

Good Luck!

1. (2p)
A LTI system is represented by the system function

$$H(z) = \frac{3 - \frac{10}{3} z^{-1}}{1 - \frac{11}{6} z^{-1} + \frac{1}{2} z^{-2}}$$

Specify the ROC of H(z) and determine h(n) for the following conditions:
a) The system is stable. (0.8p)
b) The system is causal. (0.8p)
c) The system is anticausal. (0.4p)

2. (2p)
a) Determine the frequency description and sketch the magnitude and phase function of the signal:

$$x_1(n) = 0.8\cos\left(\frac{3\pi}{5}(n-1)\right) \quad -\infty \le n \le \infty \qquad (1p)$$

b) Determine and sketch the magnitude function of the signal:

$$x_2(n) = x_1(n) \cdot w(n) \qquad \text{where } w(n) = \begin{cases} 1 & 0 \le n \le 128 \\ 0 & otherwise \end{cases} \qquad (1p)$$

Hints:
a) Fourier series expansion of a periodic discrete time signal.
b) $w(n) \cdot \cos(\omega_0 n) \leftrightarrow \frac{1}{2}\left[W(\omega - \omega_0) + W(\omega + \omega_0)\right]$.

3. (2p)
A FIR-system is described by the difference equation:

$$y(n) = \frac{1}{7}\sum_{k=0}^{6} x(n-k).$$

a) Determine the system function H(z) and sketch its pole-zero pattern. (0.8p)

b) Compute the frequency response function H(ω) of the system.
Present H(ω) as $H(\omega) = H_{real}(\omega)e^{-j\omega(M-1)/2}$ where $H_{real}(\omega)$ is a real function and M is the length of the impulse response h(n). Also sketch the magnitude- and phase-function for $-\pi \le \omega \le \pi$. (0.8p)

c) Compute the response to the input signal:

$$x(n) = 1.5 + 0.8\cos(\frac{\pi}{7}n - \frac{\pi}{7}) - 0.3\sin(\frac{4\pi}{7}n) \quad -\infty \le n \le \infty. \qquad (0.4p)$$

4. (2p)

a) Compute the linear convolution y(n)=x(n)*h(n) when:

$$h(n) = \frac{1}{3}\left[\partial(n) + \partial(n-1) + \partial(n-2)\right] \quad \text{and} \quad x(n) = u(n) - 2u(n-3) + u(n-6). \qquad (0.8p)$$

b) Compute the convolution in a) by using N-points DFT and IDFT when N=6. (0.6p)

Hint: Do the computation in the time domain.

c) An analog signal x(t) that contains a sum of three cosine signals with frequency 1200, 4200, and 6800 Hz is sampled by F_s=10 kHz.

A frequency analysis is done by DFT in N=1024 points of the windowed signal. A rectangular window of length 256 is used.

The figure below shows the magnitude of the DFT, i.e. $|X(k)|$ for $0 \le k \le 1023$.

Identify respective cosine signal in the magnitude function. (0.6p)

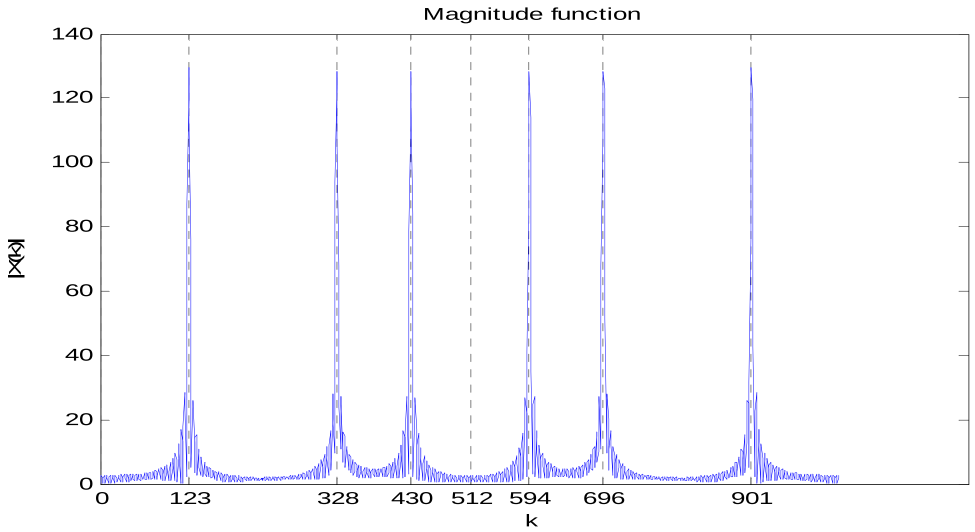

TABLE 3.3 Some Common z-Transform Pairs

	Signal, $x(n)$	z-Transform, $X(z)$	ROC
1	$\delta(n)$	1	All z
2	$u(n)$	$\dfrac{1}{1-z^{-1}}$	$\|z\| > 1$
3	$a^n u(n)$	$\dfrac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
4	$na^n u(n)$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
5	$-a^n u(-n-1)$	$\dfrac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
6	$-na^n u(-n-1)$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$
7	$(\cos\omega_0 n)u(n)$	$\dfrac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\|z\| > 1$
8	$(\sin\omega_0 n)u(n)$	$\dfrac{z^{-1}\sin\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\|z\| > 1$
9	$(a^n\cos\omega_0 n)u(n)$	$\dfrac{1-az^{-1}\cos\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\|z\| > \|a\|$
10	$(a^n\sin\omega_0 n)u(n)$	$\dfrac{az^{-1}\sin\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\|z\| > \|a\|$