Home Assignment 1 - Multivariable calculus, 2008.

Deadline: 23.09.2008.

1. Calculate the following limit (or show that it does not exist)

   (a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(2x^2+2y^2)}{\sqrt{x^2+y^2+1}-1}$. *(1/4 p)*

   (b) $\lim\limits_{(x,y)\to(2,2)} \dfrac{\ln(1+x^4-y^4)}{x^2-y^2}$. *(1/4 p)*

2. The functions $f(x,y) = x^y$, $u(x,y) = x+\ln y$, $v(x,y) = x-\ln y$ are given.
   A new function is defined as $g(u,v) = f(x(u,v),y(u,v))$.

   Calculate $\dfrac{\partial g(2,2)}{\partial u}$ and $\dfrac{\partial g(2,2)}{\partial v}$. *(1/2 p)*

3. An heat-sensitive insect is roaming the region $D:\ 0\le x\le 1,\ 0\le y\le 1$. The velocity of the insect, $\boldsymbol{v}$, is at all times in the direction in which the temperature $T(x,y) = x^2-y^2$ increases most rapidly. Furthermore, the speed $v=|\boldsymbol{v}|$ is proportional to the maximum rate of change of T with a constant of proportionality $k=\frac{1}{10}$.
   Assume that the insect is observed at the point $(\frac{1}{5},\frac{4}{5})$ at the time $t=0$.
   Where and *when* does the insect leave D ? *(1/2 p)*
   Reference: E/P 13.8 ('Directional Derivative').

4. The equation $e^{x+y+z}+z-x-y-xy=1$ implicitly defines a function $z=f(x,y)$ for which $f(0,0)=0$. Show that $(0,0)$ is a critical point. Also determine whether this point is a (local) extremum. *(1/2 p)*
   Reference: E/P 13.10.