Home Assignment 1 - Multivariable calculus, 2009.

Deadline: 25.09.2009.

1. Calculate the following limit (or show that it does not exist)

$$\lim_{(x,y)\to(0,0)} (1+2x^2+2y^2)^{\frac{1}{x^2+y^2+x^2y}}. \qquad \textit{(1/4 p)}$$

2. We are given the function

$$f(x,y) = \begin{cases} \frac{3x^2y}{x^2+y^2}\,, & (x,y) \neq (0,0) \\ 0\,, & (x,y) = (0,0) \end{cases}$$

   Show that $f_x(0,0)$ and $f_y(0,0)$ both exist.
   Also show that f_x and f_y are *not* continuous at $(0,0)$. *(1/4 p)*

3. The shape of a hill is given by $z = \dfrac{100}{1+x^2+y^2}$, where z denotes the local height (above sea-level).
   At what height is the hill steepest ? *(1/2 p)*

4. At a given instant, the length of one leg of a right triangle is $10\,cm$ and it is increasing at the rate of $2\,cm/s$.
   The length of the other leg is $12\,cm$ and it is decreasing at the rate of $3\,cm/s$. Calculate the instantaneous rate of change of the acute angle opposite the leg of length $12\,cm$. *(1/2 p)*

5. A company produces two different products. The production volumes are x and y, respectively. The running production costs are given by the function $c(x,y) = x^2 + 2xy + 2y^2$, while the retail prices (per production unit) are $p_x(x) = 10 - x$ and $p_y(y) = 20 - y$.
   How should the company adjust production volumes in order to maximize the profit ? *(1/2 p)*