

Home Assignment 2 - Multivariable calculus, 2009.

Deadline: 09.10.2009.

1. Calculate the maximum and minimum values of the function $f(x, y) = (x^2 - 3y^2) e^{-x^2 - y^2}$, where $D_f = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 4\}$.

Would the result above be different if the domain of f is changed to \mathbb{R}^2 ? (1/4 p)

2. Find the points on the curve $x^4 + x^2 y^2 + 4y^4 = 1$ that are closest to and farthest from the origin. (1/4 p)

3. Calculate the maximum and minimum values of $f(x, y, z) = 8x^2 + 4yz - 16z$ subject to the constraint $4x^2 + y^2 + 4z^2 = 16$. (1/2 p)

4. A pyramid is inscribed in the ellipsoid $\frac{x^2}{9} + y^2 + (z - 2)^2 = 1$.

The apex ('top') of the pyramid is located at the point $(0, 0, 3)$. The base area of the pyramid is a rectangle parallel to the xy -plane. Calculate the maximum volume of the pyramid.

Hint: See Ex. 3 p. 921 in EP. (1/2 p)

5. Calculate $\iint_{\Delta} \frac{(x + y)^3}{1 + (x - y)^2} dx dy$,

where Δ is a triangle with vertices at $(0, 0)$, $(1, 0)$ and $(0, 1)$. (1/4 p)

6. Calculate $\iint_D (x^2 - y^2) e^{2xy} dx dy$,

where $D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1, 0 \leq y \leq x, x \geq 0\}$. (1/4 p)