

# Home Assignment 3 - Multivariable calculus, 2008.

Deadline: 22.10.2008.

1. Find the area of the region common to the interiors of the cardioids  
 $r = 1 + \cos \theta$  and  $r = 1 - \cos \theta$ . (1/4 p)

2. Calculate  $\iint_D (x + 2y) \arctan(x - 2y) dx dy$ ,  
where  $D = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x + 2y \leq 2, 0 \leq x - 2y \leq 1\}$ . (1/4 p)

3. Calculate  $\iint_A (x^4 - y^4) e^{xy} dx dy$ , where  $A$  is the region in the first quadrant bounded by  
the curves  $xy = 1$ ,  $xy = 2$ ,  $x^2 - y^2 = 1$ , and  $x^2 - y^2 = 4$ . (1/4 p)

4. Calculate  $\iint_D (x^2 - y^2) e^{2xy} dx dy$ ,  
where  $D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1, 0 \leq y \leq x\}$ . (1/2 p)

5. Find the center of mass of a homogeneous thin plate (surface density,  $\sigma = \text{const.}$ ) which is  
bounded by  $x = e^2$ ,  $y = 0$ , and  $y = \ln x$ . (1/4 p)

6. Calculate  $\iiint_K \frac{(x^2 + y^2)z^2}{1 + x^2 + 2y^2 + 3z^2} dx dy dz$ ,  
where  $K = \{(x, y, z) \in \mathbb{R}^3 \mid 1 \leq x^2 + 2y^2 + 3z^2 \leq 3\}$ . (1/2 p)