

Tentamen i Flervariabelanalys (engelsk version)

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Minimum requirements: Grade 3: 12 p, 4: 18p, 5: 24p.

Usage of Råde/Westergren: 'Mathematics Handbook',

Papula: 'Mathematische Formelsammlung',

'Sichuan Mathematics Handbook', and calculator allowed.

1. Determine an equation of the tangent plane to the surface $2x^3 + yx - zx^2 = 3$ at a point where $x = y = -1$. (2p)

2. The function f is given by $f(x, y) = g(x - y)$, where g is a differentiable single-variable function.

Show that $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = 0$. (2p)

3. Calculate the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^2}$. (2p)

4. Determine the second order Taylor-polynomial of the function $f(x, y) = e^{x^2 - xy - y} \sqrt{1 - y}$ at the point $(0, 0)$. (3p)

5. A function f has the following directional derivatives at a point (a, b)

$$f'_{\mathbf{u}}(a, b) = 1, \quad f'_{\mathbf{v}}(a, b) = \frac{3}{5}, \quad \mathbf{u} = \frac{1}{5}(3, 4), \quad \mathbf{v} = \frac{1}{5}(-3, 4).$$

What is the maximum value of the directional derivative at this point? (3p)

6. Calculate $\int \int_D x e^{-xy} dx dy$, $D = \{(x, y) \mid 1 \leq x \leq 2, 1 \leq xy \leq 2\}$. (3p)

7. Find the largest and smallest value of the function $g(x, y) = 4x - 8xy + 2y + 1$ on the set $A = \{(x, y) \mid 0 \leq y \leq 1 - x, x \geq 0\}$. (5p)

8. Find the general solution of the following partial differential equation

$$4 \frac{\partial^2 f}{\partial x^2} - 4 \frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y^2} = 0. \quad (5p)$$

Hint: Use the following transformation $u = x + 2y$, $v = x - 2y$.

9. Calculate $\int \int \int_K x^2 dx dy dz$, $K = \{(x, y, z) \mid x^2 + y^2 \leq z \leq 2x - 4y - 4\}$. (5p)