# Multivariable calculus, 2004-10-28.

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Minimum requirements:
Comp./El. Engineering : Grade 5: 27p, 4: 21p, 3: 15p.
Information Science (ECTS) : Grade A: 27p, B: 24p, C: 21p, D: 18p, E: 15p, FX: 9p.

One of the following handbooks can be used:
Råde/Westergren: 'Mathematics Handbook', Papula: 'Mathematische Formelsammlung', Bartsch: 'Taschenbuch math. Formeln', Chaudhry/Saif-Ur-Rehman/Shahid: 'A collection of math. formulae and important results', 'Chinese math. Handbook', Spiegel/Liu: 'Schaum's Math. Handbook of Formulas and Tables', 'BIT Dhaka Collection of Formulae'.
In addition, each student can use one ordinary (non-mathematical) dictionary.

1. Find an equation of the plane tangent to the surface $z = y\,e^{x^2-y} + \dfrac{x}{y}$ at the point $(1,1,2)$. *(2p)*

2. Calculate the directional derivative of $f(x,y,z) = xy + y^3 + x\ln z$ at the point $P = (2,-1,1)$ and in the direction $\boldsymbol{u} = (1,2,2)$. *(2p)*

3. Find the following limit or show that it does not exist $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{2x^2+y^2}$. *(2p)*

4. Find all local maxima, minima, and saddle points of the function $f(x,y) = x^3 - 2y^2 + 2xy + x - 2y$. *(3p)*

5. Find the absolute minimum and maximum values of $f(x,y) = x^2 + 2y^2 + 2xy - 2x - 2y$ on the set $\Delta = \{(x,y)\,\epsilon\,\mathbb{R}^2 \mid x \geq 0\,,\ \frac{x}{2} - 1 \leq y \leq -\frac{x}{2} + 1\}$ . *(5p)*

6. The equation $z^3 + z + x^2 + xy^3 = 2$ implicitly defines a function $z = g(x,y)$ for which $g(1,-1) = 1$. Calculate $g_{xy}(1,-1)$. *(5p)*

7. Calculate $\displaystyle\iint_A e^{-(4x^2+9y^2)}\,dx\,dy\ ,\quad A = \{(x,y)\,\epsilon\,\mathbb{R}^2 |\ y \geq 0,\ 4x^2 + 9y^2 \leq 1\}$ . *(3p)*

8. Calculate $\displaystyle\iint_D \frac{1}{x^2}\ln\!\Big(\frac{y}{x}\Big)\,dx\,dy\ ,\quad D = \{(x,y)\,\epsilon\,\mathbb{R}^2 |\ 1 \leq x+y \leq 2,\ x \leq y \leq 2x\}$ . *(3p)*
   Hint: Use the following transformation $u = x+y,\quad v = \dfrac{y}{x}$.

9. Calculate $\displaystyle\iiint_K z\,dx\,dy\,dz\ ,\quad K = \{(x,y,z)\,\epsilon\,\mathbb{R}^3 |\,1 \leq x^2+y^2+z^2 \leq 4,\ z \geq \sqrt{x^2+y^2}\}$ . *(5p)*