Multivariable calculus, 2005-10-27.

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Minimum requirements:
Comp./El. Engineering : Grade 5: 27p, 4: 21p, 3: 15p.
Information Science (ECTS) : Grade A: 27p, B: 24p, C: 21p, D: 18p, E: 15p, FX: 9p.

You are allowed to use the following reference books:
Råde/Westergren: Mathematics Handbook (Beta).
One ordinary (non-mathematical) dictionary of your choice.

Note that you are not allowed to use calculators or any other electronic devices.

1. Find an equation of the plane tangent to the surface $\sqrt{x^2+y^2} - z = 1$ at the point $(3, -4, 4)$. *(2p)*

2. Find the following limit or show that it does not exist $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$. *(2p)*

3. Calculate the directional derivative of $f(x,y,z) = \ln(x^2-y) + ye^{-z}$ at the point $P = (3,1,0)$ and in the direction $\boldsymbol{v} = (2,2,1)$. *(2p)*

4. Find the (absolute) maximum and minimum values of $f(x,y) = 2x^2 + y^2 - 8x$ subject to the constraint $x^2 + y^2 = 6x$. *(3p)*

5. Calculate $\iint\limits_A \dfrac{\sin x}{x}\, dx\, dy$, where the region A is a triangle with corners at the points $(0,0), (\pi,0)$, and (π,π). *(3p)*

6. Calculate $\iint\limits_D xy\, dx\, dy$, where $D = \{(x,y)\,\epsilon\,\mathbb{R}^2 |\, 2 \le xy \le 4\,,\ 1 \le \dfrac{y}{x} \le 2,\ x > 0\}$. *(3p)*

7. Find the (absolute) minimum and maximum values of $g(x,y) = xy + x + 2y$ on the set $\Delta = \{(x,y)\,\epsilon\,\mathbb{R}^2 \mid x^2 \le y \le 2 - x\}$. *(5p)*

8. Calculate $\iiint\limits_K (x^2+y^2)\, dx\, dy\, dz$, where $K = \{(x,y,z)\ \epsilon\ \mathbb{R}^3 \mid \sqrt{\dfrac{x^2+y^2}{3}} \le z \le \sqrt{1-x^2-y^2}\}$. *(5p)*

9. Calculate $\iint\limits_S xy\sqrt{5-4z}\, dS$, where $S = \{(x,y,z)\ \epsilon\ \mathbb{R}^3 \mid z = 1 - x^2 - y^2,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$. *(5p)*