

Home Assignment 3 - Multivariable calculus, 2007.

Deadline: 16.10.2007.

1. Find the area of the region common to the interiors of the cardioids
 $r = 1 + \cos \theta$ and $r = 1 - \cos \theta$. (1/4 p)
2. Find the volume of a body which is bounded below by the xy - plane, above by the elliptic paraboloid $z = \frac{x^2}{9} + y^2$, and on the sides by the cylinder $\frac{x^2}{9} + y^2 = 1$. (1/4 p)
3. Find the center of mass of a homogeneous thin plate (surface density, $\sigma = \text{const.}$) which is bounded by $x = e^2$, $y = 0$, and $y = \ln x$. (1/4 p)
4. Calculate $\iiint_K e^{-(x^2+y^2)z^2} dx dy dz$, where $K = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 \leq 1\}$. (1/4 p)
Hint: $\int_{-\infty}^{\infty} e^{-u^2} du = \sqrt{\pi}$.
5. Calculate $\iiint_K \frac{xyz}{1 + x^2 + y^2 + z^2} dx dy dz$, where
 $K = \{(x, y, z) \in \mathbb{R}^3 \mid 1 \leq x^2 + y^2 + z^2 \leq 3, x, y, z \geq 0\}$. (1/2 p)
6. Calculate the area of the part of the sphere $x^2 + y^2 + z^2 = 1$ that lies outside the two cylinders $x^2 + (y \pm \frac{1}{2})^2 = \frac{1}{4}$. (1/2 p)