

Home Assignment 1 - Multivariable calculus, 2007.

Deadline: 18.09.2007.

1. Calculate the following limits (if they exist)

$$(a) \lim_{(x,y) \rightarrow (0,0)} \frac{\sin^3 x \sin^2 y}{1 - \cos(x^2 + y^2)},$$

$$(b) \lim_{(x,y) \rightarrow (1,1)} \frac{x^3 - y^3}{x - y},$$

$$(c) \lim_{(x,y,z) \rightarrow (0,0,0)} \frac{\ln(1 + 2(x^2 + y^2 + z^2))}{x^2 + y^2 + z^2 + \sin(xyz)}. \quad (1/2 p)$$

2. Problem 61 from chapter 13.5 in Edwards/Penney. (1/2 p)

3. A heat-seeking robot is roaming a region $D : 0 \leq x \leq 1, 0 \leq y \leq 1$ where the temperature is $T(x, y) = xy$. The velocity of the robot is given by $\mathbf{v} = \nabla T$. Assume that the robot is observed at the point $(\frac{1}{4}, \frac{1}{2})$ at the time $t = 0$.

Where and when does the robot leave D ? (1/2 p)

4. Find and classify all the critical points of the function $z(x, y)$ which is implicitly defined by the equation $e^{2zx-x^2} - 3e^{2zy+y^2} = 2$. (1/2 p)