Sensor fusion

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2009

Advantages of multi-sensor systems

- Higher signal-to-noise ratio.
- Increased reliability in the case of sensor failure.
- Reduced uncertainty and increased confidence.
- Increased discrimination power due to more comprehensive information arriving from multiple sensors.
- Improved resolution due to multiple sensors.

Difficulties

- Data in diverse formats, noisy and ambiguous
  - analogue, digital, discrete, textual, imagery
- Data dimensionality and alignment
  - Different coordinate systems, units, frequency, amplitude, timing
- Temporal alignment
  - Synchronisation of data
  - Spatial distribution of sensors demands precise time measurements
  - Data arrival at fusion node may not coincide due to variable propagation delays

Perspectives

Sensor fusion concepts can be characterized from several perspectives:

- Application domain
- Fusion objective
- Sensor type
- Sensor suite configuration
- Fusion level

Application domain

- Defense
- Robotics
- Medical
- Space
- Entertainment

Fusion objective

- Tracking of an object
- Detecting the presence of an object
- Assessing environmental conditions
- Recognizing an object

Sensor type

The major factors dictating the selection of sensors for fusion are:

- the compatibility of the sensors for deployment within the same environment
- the complementary nature of the information derived from the sensors

Sensor suite configuration

The following configurations are by far the most common:

- Parallel Configurations
- Serial Configurations
- Varied combinations of the parallel and serial configurations

The parallel configuration has been the most studied wherein the sensors are making observations concurrently.

Fusion level

- Data fusion
- Feature fusion
- decision fusion

Table: Sensor fusion in terms of input/output provided.

Mode	Example
Data in $\longrightarrow$ data out	Fusion of multi-spectral data.
Feature in $\longrightarrow$ feature out	Fusion of image and non-image data.
Decision in $\longrightarrow$ decision out	Case of non compatible sensors.
Data in $\longrightarrow$ feature out	Shape extraction.
Feature in $\longrightarrow$ decision out	Object recognition.
Data in $\longrightarrow$ decision out	Pattern recognition.

Techniques

- Artificial neural networks
- Fuzzy set theory
- Neuro-fuzzy systems
- Kernel methods (support vector machines)
- Probabilistic methods
  - Bayesian inference
  - Dempster-Shafer theory

Bayesian inference (1)

Suppose $H_1, H_2, ..., H_M$ are mutually exclusive and exhaustive hypothesis that explain the observed data S. Then

$$p(H_j|S) = \frac{p(S|H_j)p(H_j)}{\sum_i p(S|H_i)p(H_i)} \quad (1)$$

$$\sum_i p(H_i) = 1 \quad (2)$$

where $p(H_i)$ is the a priori probability of the hypothesis being true, $p(S|H_i)$ is the probability of getting the data S, given the H_i is true, and $p(H_j|S)$ is the a posteriori H_j probability given the data S.

Bayesian inference (2)

For data generated by multiple sensors $S_1, ..., S_n$

$$p(H_j|S_1 \cap ... \cap S_n) = \frac{p(H_j)[p(S_1|H_j)p(S_2|H_j)...p(S_n|H_j)]}{\sum_i p(H_i)[p(S_1|H_i)p(S_2|H_i)...p(S_n|H_i)]} \quad (3)$$

The conditional probability $p(S_i|H_j)$, the probability of observing the signal S_i, given that H_j is true, are usually determined based on experimental results.

Bayesian inference (3)

- Provides the probability of a hypothesis being true, given the evidence.
- Allows incorporating a priory knowledge of the likelihood of a hypothesis being true at all.
- The decision is usually based on the *maximum a posteriori* principle.

Dempster-Shafer theory (1)

Let $\Theta = \{\theta_1, \theta_2, ..., \theta_M\}$ be a finite set of mutually exclusive and exhaustive atomic hypotheses about some problem domain.

- The probability theory assigns *probabilities* to atomic hypotheses θ_i, while in DS theory, *a basic probability number* $m(A)$ represents one's *belief* in a not necessarily atomic hypothesis A.
- For a compound hypothesis $A \neq \theta_i$, $m(A)$ measures our belief that we are willing to commit to A. The belief cannot be subdivided amongst the subsets of A and is assigned to A at the expense of support $m(\theta_i)$.
- The belief in A and the belief in its negation $\overline{A}$ do not necessarily sum to 1.

Dempster-Shafer theory (2)

To obtain the total belief in A, (Bel(A), where $Bel : 2^\Theta \rightarrow [0,1]$), we must add the basic probability numbers $m(B)$ for all subsets B of A.

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{4}$$

- The subsets B of Θ for which $m(B) > 0$ are called the *focal elements* of the belief function.
- The union of the focal elements is called the *core* of the belief function.
- The belief functions having only one focal element in addition to Θ are called *simple support functions*.

Dempster-Shafer theory (3)

Given two basic probability assignments m_1 and m_2 associated with Bel_1 and Bel_2 induced by two different sources of information over the same Θ can be combined if their cores are not disjoint. The *orthogonal sum* ($m = m_1 \bigoplus m_2,\ m : 2^{\Theta} \rightarrow [0,1]$) is a convenient way to perform such a combination:

$$m(\emptyset) = 0 \quad (5)$$

$$m(A) = K^{-1} \sum_{B \cap D = A} m_1(B) m_2(D),\ A \neq \emptyset \quad (6)$$

$$K = \sum_{B \cap D \neq \emptyset} m_1(B) m_2(D) \quad (7)$$

Dempster-Shafer theory (4)

- The function m is a basic probability assignment.
- The core of Bel given by m equals the intersection of the cores of Bel_1 and Bel_2.
- The combination rule is commutative and associative, and it may be generalized to combine multiple evidences.
- Maximum belief over the atomic hypothesis is the most commonly used approach to make decisions based on the DS fusion result.

Fuzzy set theory (1)

- The advantages of the fuzzy set theory lie in the variety of combination operators.
- Fusion by IF-THEN rules, for example

$$\texttt{IF } x_1 \texttt{ is } A_1 \texttt{ AND } \ldots \texttt{ AND } x_n \texttt{ is } A_n \texttt{ THEN class } C_q \texttt{ with } z^q \quad (8)$$

where $A_i (i = 1, \ldots, n)$ are fuzzy sets defined over the input variables x_i, C_q is a class label and z^q is a rule weight. Each fuzzy set is represented by a membership function, for example Gaussian:

$$\mu_i = \exp\left(-\frac{(x_i - c_i)^2}{\sigma_i^2}\right) \quad (9)$$

where c_i and σ_i are the center and the width of the Gaussian function, respectively.

Fuzzy set theory (2)

- Fuzzy integration is an example of a successful combination operator.
- The fuzzy integral is interpreted as searching for the maximal grade of agreement between the objective evidence and the expectation.
- A non additive fuzzy measure (FM) is used in the fuzzy, for example Choquet, integral.

Fuzzy set theory (3)

If we assume that $Z = \{z_1, z_2, \ldots, z_L\}$ is a non-empty finite set of data from L sources/sensors and g is an FM on Z, the *discrete Choquet integral* of a function $h : Z \rightarrow \mathbb{R}^+$ with respect to g is defined as

$$C_g(h(z_1), \ldots, h(z_L)) = \sum_{i=1}^{L} [h(z_i) - h(z_{i-1})] g(A_i) \qquad (10)$$

where indices i have been permuted so that
$0 \leq h(z_1) \leq ... \leq h(z_L) \leq 1$; $A_i = \{z_i, ..., z_L\}$; $h(z_0) = 0$.

- If decision fusion is considered, $h(z_i)$ stands for the classification result of the ith classifier.

Fuzzy set theory (4)

A set function $g : 2^Z \rightarrow [0, 1]$ is a *fuzzy measure* if

- 1) $g(\emptyset) = 0$; $g(Z) = 1$;
- 2) *if* $A, B \subset 2^Z$ and $A \subset B$ then $g(A) \leq g(B)$;
- 3) *if* $A_n \subset 2^Z$ for $1 \leq n < \infty$ and $\{A_n\}$ is monotonic in the sense of inclusion, then $\lim_{n\rightarrow\infty} g(A_n) = g(\lim_{n\rightarrow\infty} A_n)$.

Fuzzy set theory (5)

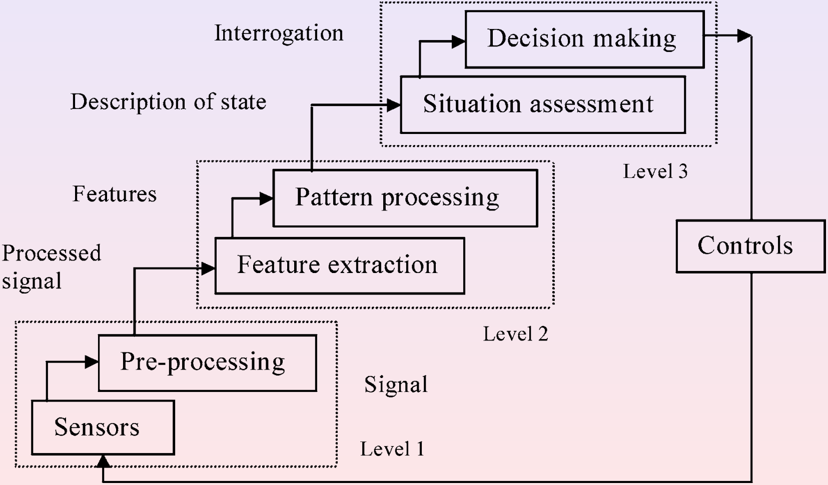
An ordinary FM of a union of two disjoint subsets cannot be directly computed from the FMs of the subsets. The so called λ-FM allows such computation:

$$g(A \cup B) = g(A) + g(B) + \lambda g(A)g(B) \tag{11}$$

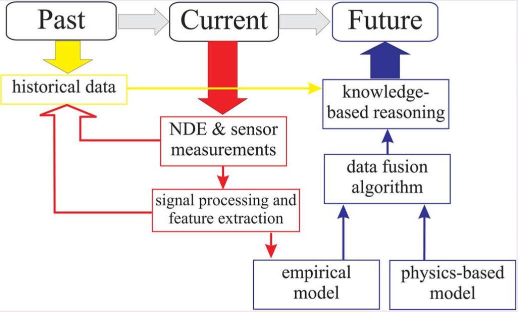
When g is the λ-FM, the values of $g(A_i)$ can be computed recursively. If $g^i = g(\{z_i\})$, then

- $g(A_1) = g(\{z_1\}) = g^1$
- $g(A_i) = g^i + g(A_{i-1}) + \lambda g^i g(A_{i-1})$, for $1 < i \leq L$.

A waterfall fusion model



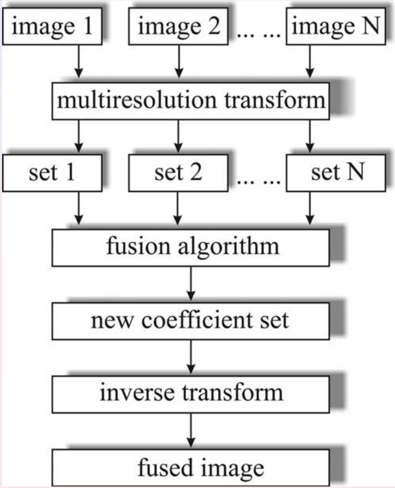
Diagnostic-prognostic monitoring



- The prediction of future is based on the knowledge of current conditions, the physical model of a process, and past events. (NDE=nondestructive evaluation).

- At the signal level, the fusion of redundant information enhances the reliability of the system.
- At a higher level, the fusion of physical and numerical models provides an input to the diagnostic and prognostic reasoners.

Multiresolution image fusion (1)



Two aspects of the fusion scheme:

- Multiresolution algorithms for image decomposition and reconstruction.
- The combination rule for the coefficient sets in the transform domain.

Multiresolution image fusion (2)

- The input image is decomposed into a set of spatial frequency bandpass sub-images.
- An image $I(x, y)$ can be represented as a sum over a collection of basic functions $g_i(x, y)$

$$I(x, y) = \sum_i y_i(x, y) g_i(x, y) \tag{12}$$

  where i is the resolution level and $y_i(x, y)$ are the transform coefficients.
- The coefficients can be computed by projecting the image onto a set of projection functions $h_i(x, y)$:

$$y_i(x, y) = \sum_{x,y} h_i(x, y) I(x, y) \tag{13}$$

- The functions $h_i(x, y)$ are the translated and dilated copies of one another.