Exam in Signal analysis and representation, 7.5 credits.
Course code: dt8010
Date: 2010-01-04

Allowed items on the exam:
Tables of Signal processing formulas.
Tables of Mathematical formulas.
Calculator.

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Maximum points: 8.
In order to pass the examination with a grade 3 a minimum of 3.3 points is required.
To get a grade 4 a minimum of 4.9 points is required, and to get a grade 5 a minimum of 6.5 points is required.

Give your answer in a readable way and motivate your assumptions.

Good Luck!

1. (2p)
An LTI system is described by the difference equation

$$y(n) = x(n) + 2x(n-1) + x(n-2)$$

a) Determine the impulse response h(n) and motivate if the system is stable or not. (0.6p)
b) Determine and sketch the frequency response function $H(\omega)$ for $0 \le \omega \le 2\pi$.
Present $H(\omega)$ as $H(\omega) = H_{real}(\omega)e^{-j\omega(M-1)/2}$ where $H_{real}(\omega)$ is a real function and M is the length of the impulse response h(n). (0.7p)
c) Now consider a new system $H_1(\omega)=H(\omega+\pi)$. Determine the impulse response $h_1(n)$ of the new system. (0.7p)

2. (2p)
a) Determine the frequency description and sketch the magnitude and phase function of:

$$x_1(n) = 0.5\cos\left(\frac{\pi}{3}n\right) + 0.8\sin\left(\frac{\pi}{5}n\right) \quad -\infty \le n \le \infty . \quad \text{(1p)}$$

b) Sketch the magnitude function for $0 \le \omega \le \pi$ of:

$$x_2(n) = x_1(n)\cdot w(n) \quad \text{where } w(n) = \begin{cases} 1 & 0 \le n \le 255 \\ 0 & otherwise \end{cases} . \quad \text{(1p)}$$

Hints:
a) Fourier series expansion of a periodic discrete time signal.
b) $w(n)\cdot\cos(\omega_0 n) \leftrightarrow \frac{1}{2}\left[W(\omega-\omega_0) + W(\omega+\omega_0)\right]$

$w(n)\cdot\sin(\omega_0 n) \leftrightarrow \frac{1}{2j}\left[W(\omega-\omega_0) - W(\omega+\omega_0)\right].$

3. (2p)
An LTI system is represented by the system function

$$H(z) = \frac{0.1(1-z^{-2})}{1-0.9z^{-1}+0.81z^{-2}}$$

a) Sketch the pole-zero pattern and the magnitude of the frequency response function $|H(\omega)|$ for $-\pi \le \omega \le \pi$. (1p)
b) Compute the response to the input signal:

$$x(n) = 0.4 + 0.4\cos\left(\frac{\pi}{3}(n-2)\right) \quad -\infty \le n \le \infty . \quad \text{(1p)}$$

4. (2p)
a) An analog signal x(t) that contains a sum of three cosine signals with frequency 1500, 4600, and 5800 Hz is sampled by F_s=8 kHz. A frequency analysis is done by DFT in N=2048 points of the windowed signal. A rectangular window of length 256 is used.
Sketch the magnitude of the DFT, i.e. $|X(k)|$. The frequency axis should be graded in k. (0.6p)

b) Select a sample frequency to avoid the aliasing effect when sampling x(t). A frequency analysis is done by DFT (N=2048, rectangular window of length=256).
Sketch the magnitude of the DFT of the "aliasing-free" discrete time signal (the frequency axis should be graded in k). (0.6p)

c) Compute the linear convolution y(n)=x(n)*h(n) using N-points DFT and IDFT when:

$h(n) = \frac{1}{2}\left[\partial(n) - \partial(n-1)\right]$ and $x(n) = -u(n) + 2u(n-3) - u(n-6)$. (0.8p)