LÖSNINGSFÖRSLAG TILL TENTAMEN I ELLÄRA 100525

1.

$$u_1(t) = 3\sin(\omega t - 30^\circ) + 3 \quad [\text{Volt}]$$

$$u_2(t) = 1\sin(\omega t + 45^\circ) \quad (\text{Volt})$$

då $\omega = 30$ rad/sek

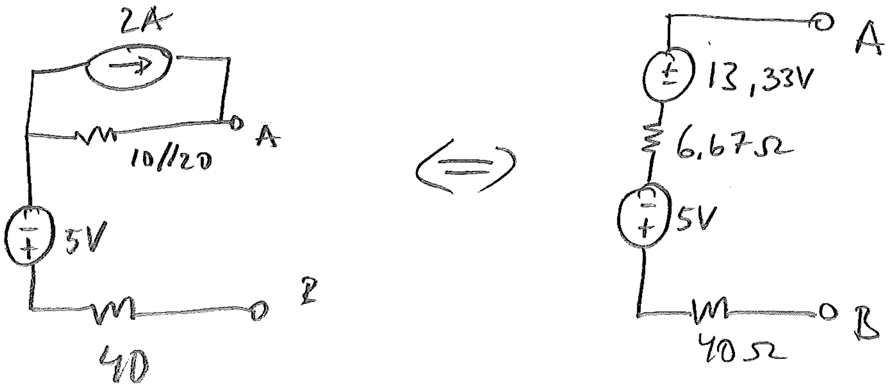
$$U_{RMS} = \sqrt{U_{ACe}^2 + U_{DC}^2}$$

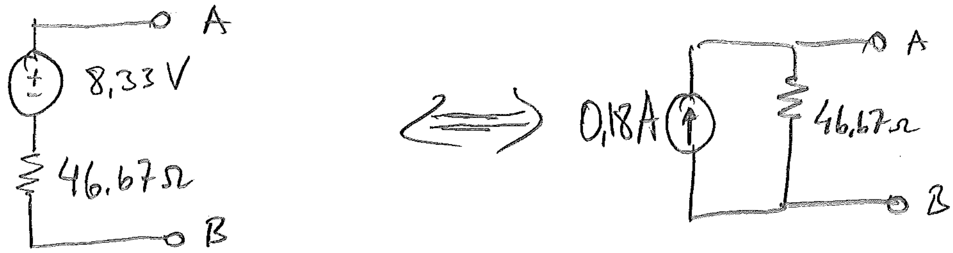
$$U_{1eff} = \sqrt{\left(\frac{3}{\sqrt{2}}\right)^2 + 3^2} \approx 3{,}67$$

$$U_{2eff} = \frac{1}{\sqrt{2}} = 0{,}71 \text{ Volt}$$

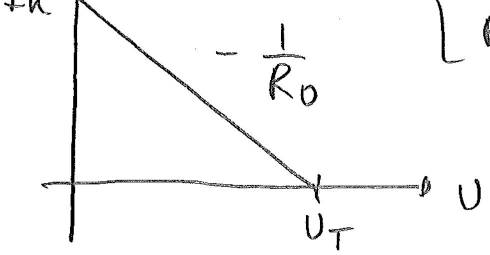
2.

a)





b)



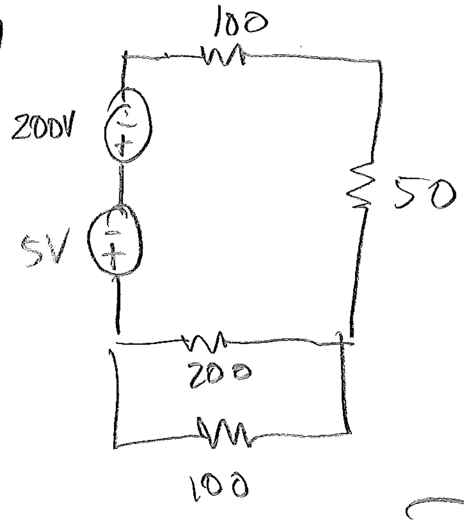
$$\begin{cases} U_T = 8{,}33 \text{ V} \\ I_K = 0{,}18 \text{ A} \\ R_0 = 46{,}67\ \Omega \end{cases}$$

c) $R_L = R_0 = 46{,}67$ ger anpassning och max effekt ut.

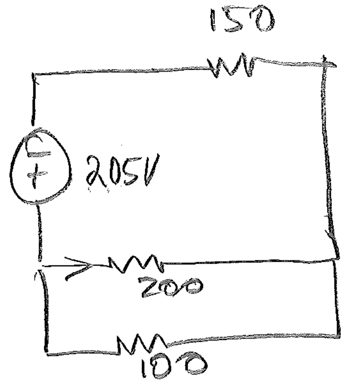
$$P = \frac{(8{,}33/2)^2}{R_L} = 0{,}372 \text{ W}$$

Tvåpolsomvandla! först.

3. a)

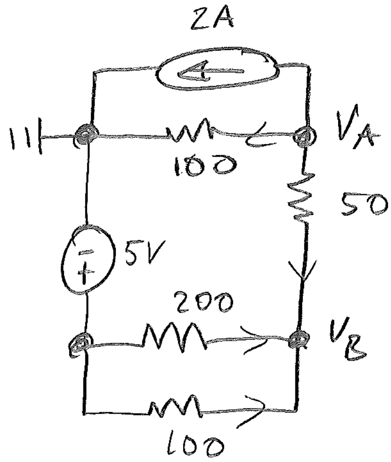


$\Longleftrightarrow$



$$I_{200} = \underbrace{\frac{205\,V}{\underbrace{200/\!/100}_{66,7} + 150}}_{\text{total ström ut från 195V-källan.}} \cdot \overbrace{\frac{\frac{1}{200}}{\frac{1}{200}+\frac{1}{100}}}^{\text{strömdelning}} \approx 0,315\ A$$

b)



Det blir endast 2 obekanta nodpotentialer. (Se figur!)

Ansätt strömmar därefter. KCL i punkter A och B.

nod A: "in" = "ut"

$$0 = 2 + \frac{(V_A - 0)}{100} + \frac{V_A - V_B}{50}$$

$$200 + V_A + 2V_A - 2V_B = 0$$

$$\frac{200 + 3V_A}{2} = V_B$$

nod B: "in" = "ut"

$$\frac{(V_A - V_B)}{50} + \frac{(5 - V_B)}{200} + \frac{5 - V_B}{100} = 0$$

$$4V_A - 4V_B + 5 - V_B + 10 - 2V_B = 0$$

$$4V_A + 15 = 7V_B$$

$$\frac{4V_A + 15}{7} = V_B$$

Lös ut V_B!

$$\frac{200 + 3V_A}{2} = \frac{4V_A + 15}{7}$$

$$1400 + 21V_A = 8V_A + 30$$

$$13V_A = -1370$$

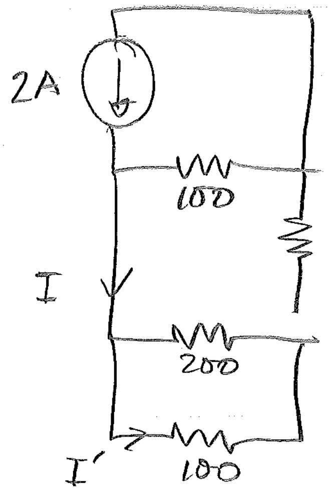
$$V_A = -\frac{1370}{13} \approx -105,4\,V$$

$$V_B = -58,1\ V$$

$$U_{R8} = 5 - (-58,1) = 63,1\ V$$

3.c "Superposition": börja med 2A-källan.

Fall 1

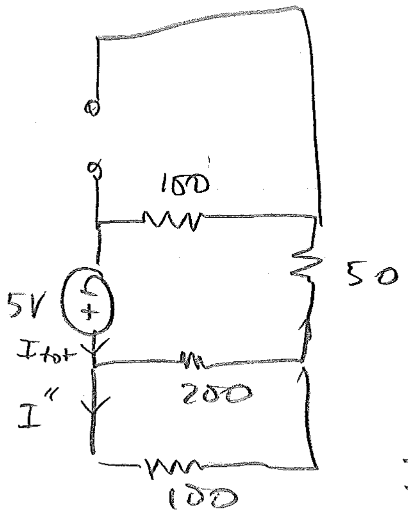


$$I = 2A \cdot \frac{\frac{1}{200//100+50}}{\frac{1}{200//100+50} + \frac{1}{100}} \approx 2 \cdot \frac{1}{1+1{,}167}$$

$$I \approx 0{,}923 A$$

$$I' = I \cdot \frac{\frac{1}{100}}{\frac{1}{100} + \frac{1}{200}} = I \cdot \frac{1}{1+\frac{1}{2}} \approx 0{,}6153 A$$

Fall 2



$$I_{tot} = \frac{5V}{150 + 200//100} \approx 0.0231 A$$

$$I'' = I_{tot} \cdot \frac{\frac{1}{100}}{\frac{1}{100} + \frac{1}{200}} \approx I_{tot} \cdot \frac{1}{1+\frac{1}{2}} \approx 0{,}0154 A$$

$$I_{R8} = 0.6307 A$$

$$U_{R8} = R_8 \cdot I_{R8} \approx \underline{63{,}1 V}$$

4.

$$I_L = \frac{12e^{j0°}}{j\omega L} = \frac{12e^{j0°}}{10\,e^{j90°}} = 1,2\,e^{-j90°}$$

$$I_R = \frac{12\,e^{j0°}}{20} = 0.6\,e^{j0°}$$

$$I = I_L + I_R = 1,2\,(\cancel{\cos(-90°)} + j\sin(-90°)) + 0.6$$

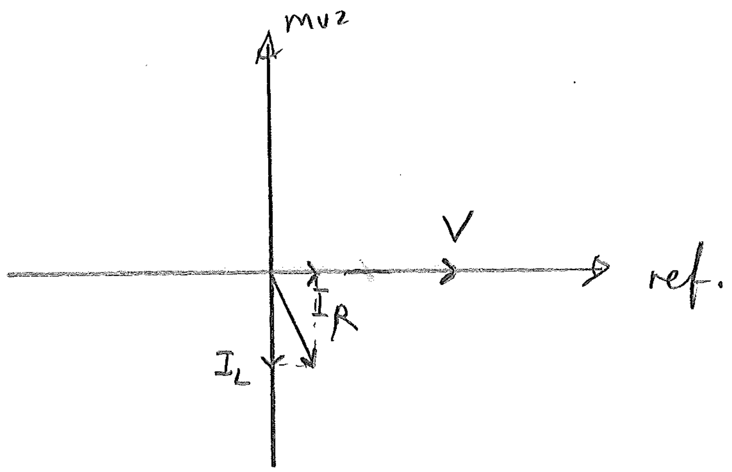
$$= 0,6 - j1,2 = 1,34\,e^{-j63,4°}$$

$$i(t) = 1,34\sin(1000t - 63,4°)\ [A]$$

skalor

$\longmapsto$ 1A

$\longmapsto$ 5V



5.

a)

$$U_{ut}(j\omega) = U_{in}(j\omega)\cdot\frac{R_4 /\!/ j\omega L_3}{R_4 /\!/ j\omega L_3 + \frac{1}{j\omega C_1}} = U_{in}\,\frac{\frac{R_4\cdot j\omega L_3}{R_4 + j\omega L_3}}{\frac{R_4\cdot j\omega L_3}{R_4 + j\omega L_3} + \frac{1}{j\omega C_1}} =$$

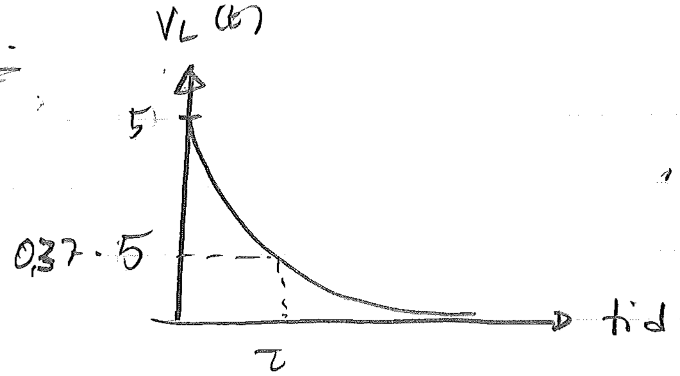
$$= U_{in}\cdot\frac{R_4\cdot j\omega L_3}{R_4\cdot j\omega L_3 + \frac{R_4 + j\omega L_3}{j\omega C_1}} = U_{in}\cdot\frac{(j\omega)^2\cdot C_1\cdot L_3\cdot R_4}{(j\omega)^2\cdot C_1\cdot L_3\cdot R_4 + j\omega L_3 R_4}$$

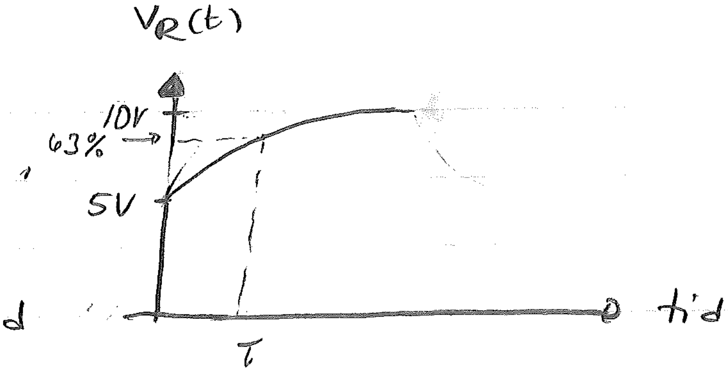
b) $\omega \to 0$: $\quad H(j\omega) = \frac{U_{ut}}{U_{in}}(j\omega) \longrightarrow 0$

$\omega \to \infty$: $\quad H(j\omega) = \frac{U_{ut}}{U_{in}}(j\omega) \to 1$

Aha ett högpassfilter!

6.

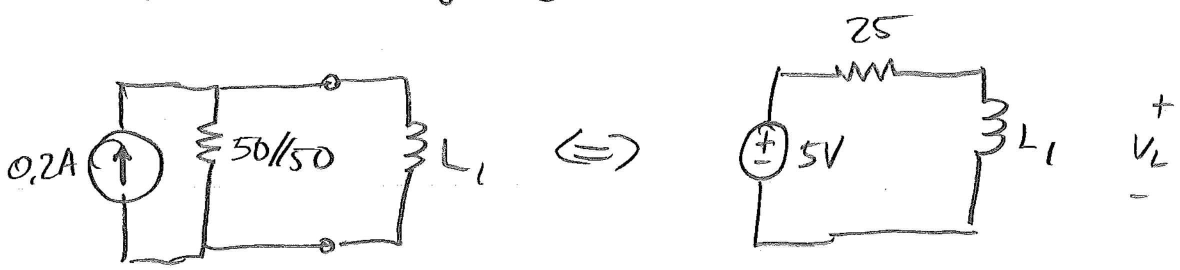




efter ett tag blir induktorn som en kortslutning och detta betyder att R10 blir betydelselös. På vägen dit bestäms τ av båda resistorerna.

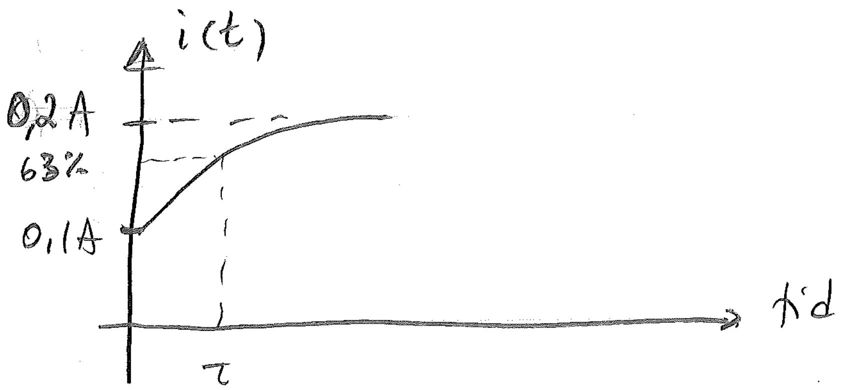
Tvåpolsomvandling ger:

$$\tau = \frac{L}{R} = \frac{100m}{25} = 4ms$$



$$v_L(t) = 5 e^{-t/\tau}$$

$$V_R(t) = 5 + 5\left(1 - e^{-t/\tau}\right)$$



$$i(t) = 0,1 + 0.1\left(1 - e^{-t/\tau}\right)$$

$\underline{\underline{7.}}$ Spänning på sek. sidan blir: $\hat{U}_2 = \hat{U}_1 \frac{N_2}{N_1}$

a) $\hat{U}_2 = 15 \cdot \frac{200}{700} \approx 4{,}29$ Volt $\quad u_2(t) = 4{,}29 \sin(200t)$ [V]

$$Z = 5 + 3j \approx \sqrt{5^2 + 3^2} \cdot e^{j31^\circ} [\Omega] = 5{,}83\, e^{j31^\circ} (\Omega)$$

$$I_2 = \frac{U_2}{Z} = \frac{4{,}29\, e^{j0^\circ}}{5{,}83\, e^{j31^\circ}} = 0{,}74\, e^{-j31^\circ} (A)$$

$$P = R \cdot I_{2e} = 5 \cdot \left(\frac{0{,}74}{\sqrt{2}}\right)^2 = 1{,}37 \text{ W}$$

b) $Q = X \cdot I_{2e}^2 = 3 \cdot \left(\frac{0{,}74}{\sqrt{2}}\right)^2 = 0{,}82$ VAr

c) $\frac{\hat{I}_2}{\hat{I}_1} = \frac{N_1}{N_2} \Rightarrow \hat{I}_1 = \hat{I}_2 \frac{N_2}{N_1} = 0{,}74 \cdot \frac{200}{700} = 0{,}21$ A

$$i_1(t) = 0{,}21 \sin(200t - 31^\circ)$$

$\underline{\underline{8.}}$ $\frac{1}{Z_{ers}} = \frac{1}{Z_1} + \frac{1}{Z_2} \Rightarrow Z_{ers} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$

$$Z_1 = 15 - j8 = \sqrt{15^2 + 8^2}\, e^{-j28{,}1^\circ} = 17\, e^{-j28{,}1^\circ} (\Omega)$$

$$Z_2 = 25 + j4 = \sqrt{25^2 + 4^2}\, e^{j9{,}1^\circ} \simeq 25{,}3\, e^{j9{,}1^\circ} [\Omega]$$

$$Z_{ers} = \frac{17 \cdot e^{-j28{,}1^\circ} \cdot 25{,}3\, e^{j9{,}1^\circ}}{15 - j8 + 25 + j4} = \frac{430{,}1\, e^{-j19^\circ}}{40 - j4} \approx \frac{430{,}1\, e^{-j19^\circ}}{\sqrt{40^2 + 4^2}\, e^{-j5{,}7^\circ}}$$

$$\approx 10{,}7\, e^{-j13{,}3^\circ} [\Omega] \approx 10{,}41 - j2{,}46\, [\Omega]$$

9.

$\underline{Z_1} = 100{,}5\, e^{-j5{,}7^\circ}$

$\underline{Z_2} = 79{,}1\, e^{j18{,}4^\circ}$

$\underline{Z_3} = 94{,}9\, e^{j71{,}6^\circ}$

eff.värde

a) $I_1 = \frac{U}{Z_1} = \frac{230\, e^{j0^\circ}}{100{,}5\, e^{-j5{,}7^\circ}} \approx \underline{2{,}29\, e^{j5{,}7^\circ}}$

$I_2 = \frac{U}{Z_2} = \frac{230 e^{j0^\circ}}{79{,}1 e^{j18{,}4^\circ}} \approx \underline{2{,}91\, e^{-j18{,}4^\circ}}$ | $I_3 = \frac{U}{Z_3} = \frac{230\, e^{j0^\circ}}{94{,}9\, e^{j71{,}6^\circ}} \approx \underline{2{,}42\, e^{-j71{,}6^\circ}}$

b) $I = I_1 + I_2 + I_3 = 2{,}29\,(\cos 5{,}7^\circ + j \sin 5{,}7^\circ) + 2{,}91\big((\cos -18{,}4^\circ) + j \sin(-18{,}4^\circ)\big)$
$+\ 2{,}42\,(\cos(-71{,}6^\circ) + j \sin(-71{,}6^\circ)) = 5{,}80 - j2{,}99$
$\approx \underline{6{,}52\, e^{-j27{,}3^\circ}}$ [A]

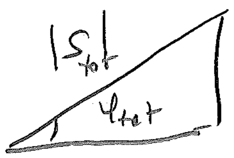
c) $P_1 = R_1 I_{1e} = 100 \cdot 2{,}29^2 = 524{,}4$ W
$P_2 = R_2 I_{2e} = 75 \cdot 2{,}91^2 = 635{,}1$ W
$P_3 = R_3 \cdot I_{3e} = 30 \cdot 2{,}42^2 = 175{,}7$ W

$P_{tot} = P_1 + P_2 + P_3 =$
$= \underline{1335{,}2}$ W

d) $|S_1| = U \cdot I_{1e} = 526{,}7$ VA , $|S_2| = U \cdot I_{2e} = 669{,}3$ VA
$|S_3| = U \cdot I_{3e} = 556{,}6$ VA

notera: $|S|_{tot} \neq |S_1| + |S_2| + |S_3|$
däremot $|S_{tot}| = U \cdot I_e = 230 \cdot 6{,}52 \approx \underline{1500}$ VA

e) "före"

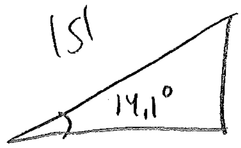


$Q_{tot} = 683{,}3$ VAr

$P_{tot} = P_1 + P_2 + P_3 = 1335{,}2$ W

$\varphi_{tot} = 27{,}1^\circ$

"efter"



$Q_{tot} + Q_C$

$\cos\varphi = 0{,}97$
$\varphi = 14{,}1^\circ$

$P_{tot} = 1335{,}2$

$\tan\varphi = \frac{Q_{tot} + Q_C}{P_{tot}}$

$Q_{tot} + Q_C = 335{,}4$ VAr

$Q_C = -348$ VAr $= -U_e^2 \cdot \omega C \Rightarrow C \approx 20{,}9\ \mu$

$|S_{tot}| = \sqrt{P_{tot}^2 + Q_{tot}^2} = U_e \cdot I_e$

$\rightarrow I_E \approx 5{,}98$ A

f) $P_{tot} = R \cdot I_e^2 = 1335{,}2$ W
$Q_{tot} + Q_C = X \cdot I_e^2 = 335{,}4$ VAr
$\rightarrow X \approx 9{,}4$
$\rightarrow R \approx 37{,}3\,\Omega$
$Z = \underline{37{,}3} + j\underline{9{,}4}$ [Ω]