

## Test-exam Single Variable Calculus.

Grade Limits: Grade 3: 12p, 4: 18p, 5: 24p.

Note: No calculators or computers allowed.

1. Find the following limit or show that it does not exist  $\lim_{x \rightarrow \infty} \frac{1}{\sqrt{x^2 + 3x - x}}$  . (2p)
  
2. Find an equation for the tangent to the curve  $y = \arctan(x^3)$   
at the point with  $x$  - coordinate 1. (3p)
  
3. Calculate
  - a)  $\int_1^3 \frac{5x + 9}{x^2 + 4x + 3} dx$  , (3p)
  - b)  $\int_0^2 x^2 \sqrt{x^3 + 1} dx$  , (3p)
  - c)  $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x + 1} dx$  . (3p)
  - d)  $\int x (\ln x)^2 dx$  . (3p)
  
4. Calculate the largest possible area of a triangle with vertices (corners) at the points  
(0, 0), (x, 0) and (x, e<sup>-2x<sup>2</sup></sup>) ( $x \geq 0$ ). (3p)
  
5. Calculate all local extrema, zeroes (solutions of the equation  $f(x) = 0$ ) and asymptotes  
of  $f(x) = \frac{x^2 - x}{x - 2}$  .  
Also sketch the graph of  $f$  together with the asymptotes. (5p)
  
6. Calculate the volume of the body which is obtained by rotating the curve  
 $y = e^{x^2} \sqrt{x}$ ,  $0 \leq x \leq 1$  about the  $x$ -axis. (5p)