

Test-exam Single Variable Calculus.

Grade Limits: Grade 3: 12p, 4: 18p, 5: 24p.

Note: No calculators or computers allowed.

1. Find the following limit or show that it does not exist $\lim_{x \rightarrow \infty} \frac{1}{\sqrt{x^2 + 3x - x}}$. (2p)
2. Find an equation for the tangent to the curve $y = \arctan(x^3)$ at the point with x - coordinate 1. (3p)
3. Calculate
 - a) $\int_1^3 \frac{5x + 9}{x^2 + 4x + 3} dx$, (3p)
 - b) $\int_0^2 x^2 \sqrt{x^3 + 1} dx$, (3p)
 - c) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x + 1} dx$. (3p)
 - d) $\int x (\ln x)^2 dx$. (3p)
4. Calculate the largest possible area of a triangle with vertices (corners) at the points $(0, 0)$, $(x, 0)$ and (x, e^{-2x^2}) ($x \geq 0$). (3p)
5. Calculate all local extrema, zeroes (solutions of the equation $f(x) = 0$) and asymptotes of $f(x) = \frac{x^2 - x}{x - 2}$.
Also sketch the graph of f together with the asymptotes. (5p)
6. Calculate the volume of the body which is obtained by rotating the curve $y = e^{x^2} \sqrt{x}$, $0 \leq x \leq 1$ about the x -axis. (5p)