Exercise 2

Stability; Frequency responses; Mikhaylov and Nyquist curves

1. The Mikhaylov curve $A(e^{-i\omega})$, $\omega = 0 \to 2\pi$, can easily be calculated by use of Sysquake or Matlab as follows. Given the characteristic polynomial $A(q^{-1}) = 1 - 0.5q^{-1} - 4q^{-2} + 2q^{-3}$, the frequency curve can be plotted by the commands below (notice the dot-operation that gives element-wise operation).

```
w = [0:0.01:2*pi];
z = exp(-i*w);
Az = 1 - 0.5*z - 4*z.^2 + 2*z.^3;
plot(real(Az),imag(Az));
```

   A figure then shows a curve making two revolutions around the origin, i.e. there are two unstable poles. These can also be directly calculated by the commands

```
A = [1 -0.5 -4 2];
roots(A)
```

   which produce the display of $2, -2, 0.5$. Determine stability/instability for the following characteristic polynomials by evaluating $A(e^{-i\omega})$, $\omega = 0 \to 2\pi$ as well as calculating the poles.

   a. $A(q^{-1}) = 1 - 1.5q^{-1} + 0.9q^{-2}$

   b. $A(q^{-1}) = 1 - 3q^{-1} + 2q^{-2} - 0.5q^{-3}$

   c. $A(q^{-1}) = 1 - 2q^{-1} + 2q^{-2} - 0.5q^{-3}$

   d. $A(q^{-1}) = 2 - 3q^{-1} + 1.8q^{-2}$

   e. $A(q^{-1}) = 1 + 5q^{-1} - 0.25q^{-2} - 1.25q^{-3}$

   f. $A(q^{-1}) = 1 - 1.7q^{-1} + 1.7q^{-2} - 0.7q^{-3}$

2. Given the process

$$y(k) = G(q^{-1})u(k), \quad G(q^{-1}) = \frac{0.2q^{-5}}{1 - q^{-1} + 0.5q^{-2}}$$

   a) Choose the input $u(k) = cos(0.1k)$ and calculate both the real response $y(k)$ (use the function filter) and the asymptotic response $y_a(k) = |G(e^{-i0.1})| \cos(0.1k + \arg G(e^{-i0.1}))$. Plot both these together to verify that $y(k) \to y_a(k)$.

   b) Suppose that proportional (negative) feedback $u(k) = -Ky(k)$ is to be used. Plot the Nyquist curve $G(e^{-i\omega}), \omega = 0 \to \pi$ and use this to find the gain margin (maximum gain $K_{\max}$ for which the closed-loop system is stable). Verify $K_{\max}$ by calculating the corresponding closed-loop poles for this gain. Two poles should be on the stability margin. What is the argument for this pole pair? Compare with the ω for which the Nyquist curves crosses the negative axis!

3. Consider the process (see seminar notes!)

$$G(q^{-1}) = \frac{0.1q^{-2}}{(1 - 0.1q^{-1})(1 - 0.7q^{-1})(1 - 0.9q^{-1})}$$

   a) Plot the Nyquist curve and estimate the amplitude (gain) and phase margins.

   b) From the phase margin, estimate the delay margin. Thus, how many sample delays can be introduced before the closed-loop gets unstable? Verify the maximum delay by introducing it and calculating the corresponding closed-loop poles. Two of these should then be on the stability margin. What is the argument for this pole pair? Compare with the ω for which the Nyquist curves crosses the unit circle!