Exercise 4

Pole-placement design for level regulation of a tank

In this exercise, **Sysquake** will be used to study the control design of a water filling tank process. Download the Sysquake file `Tank.sq` and open it in **Sysquake**. A tank process will then be displayed graphically. The input to the process is the water flow (equivalently the pump voltage) and the output is the level in the tank which is used for digital feedback control. If there were no outflow from the tank, the dynamics would be an 'integrator'. Here, however, there is a small outflow which makes the system stable. But since the outflow is much smaller than the inflow, it can be neglected in comparison. A crude approximation of the process dynamics can therefore be described by

$$y(k) = y(k-1) + u(k-1)$$

where y is the sampled level in the tank and u the input water flow (pump speed). In polynomial form, the system is

$$A(\mathrm{q}^{-1})y(k) = B(\mathrm{q}^{-1})u(k), \quad A(\mathrm{q}^{-1}) = 1 - \mathrm{q}^{-1}, \quad B(\mathrm{q}^{-1}) = \mathrm{q}^{-1}$$

Manual control

To get some feeling for how the tank process dynamics reacts for different inputs, try to first control the process manually. The control signal used is displayed in the first window as a P-controller with gain K and a bias level u_0. The value u_0 can be used to set the equilibrium point.

- Choose the gain of the P-controller $K = 0$ to turn off the proportional feedback. The process in now running in open loop. It will react upon your manual input from u_0. Try to fill the tank by increasing u_0 and avoid getting overflow! The input is limited to a maximum flow and also the minimum flow is zero (the pump cannot suck up water).

- Try to find the equilibrium u_0 corresponding to the reference level mark as a red arrow at the tank. Then after the equilibrium is reached, click on the red valve to open it. More water is then flowing out of the tank and you must then compensate this disturbance by increasing u_0.

P-control

- Close the valve and choose proportional feedback by increasing K. The control signal is now $u = u_0 + K(r - y)$. The difference equation description considers deviation from the equilibrium corresponding to u_0, thus where the input (deviation from u_0) is $u = K(r - y)$. The closed-loop characteristic polynomial is then $A_c = A + BK$ and the closed loop is

  $$y(k) = \frac{BK}{A + BK} r(k) = \frac{K\mathrm{q}^{-1}}{1 - (1-K)\mathrm{q}^{-1}} r(k)$$

  Choose *dead-beat* control $K = 1$ to make the output track the reference with just one sample delay. Verify this behavior by making a step change in the reference level r (click and move the red arrow). The step must of course be small enough not to cause the control signal to change.

- Choose different gains to verify that the closed-loop system behaves like a first order system. The response should be monotonous for $K < 1$ and starting to oscillate for $K > 1$ (why?).

- Adjust u_0 to make $y = r$, then open the valve. This can be considered as a step disturbance entering somehow at the process. Thus, the model is

  $$\begin{aligned} Ay &= Bu + Cd \\ u &= K(r - y) \end{aligned} \qquad \rightarrow y = \frac{BK}{A_c} r + \frac{C}{A_c} d$$

  As a result of the disturbance step (opening of the valve) there will be a bias (stationary error).

Integrating control

- Click in the first window in the choice for integral action. Also, unmark *antiwindup*. The controller is then changed to

$$R(q^{-1})u(k) = -S(q^{-1})y(k) + T(q^{-1})r(k), \quad \begin{cases} R(q^{-1}) = 1 - q^{-1} \\ S(q^{-1}) = K(1 - zq^{-1}) \\ T(q^{-1}) = S(1) = K(1 - z) \end{cases}$$

  or equivalently in recursive form

$$u(k) = u(k-1) - Ky(k) + (Kz)y(k-1) + K(1-z)r(k)$$

  The controller pole is at 1 (integral action) and it also has a zero at z. The parameterization of the controller makes it possible to change (interactively) the gain K and the zero z. Notice that u_0 is not needed because of the integral action. The closed-loop characteristic polynomial is

$$A_c = AR + BS = (1 - q^{-1})^2 + q^{-1}K(1 - zq^{-1}) = 1 - (2 - K)q^{-1} + (1 - Kz)q^{-2}$$

  Dead-beat design corresponds to placing both poles at the origin, i.e. $A_c = (1 - \lambda_1 q^{-1})(1 - \lambda_2 q^{-1}) = 1$. This is achieved here by choosing

$$\begin{cases} K = 2 \\ z = 1/2 \end{cases}$$

  The closed-loop response is for this tuning

$$y(k) = \frac{BT}{A_c}r(k) + \frac{CR}{A_c}d(k) = r(k-1) + C[d(k) - d(k-1)]$$

  Thus, a step disturbance d will be eliminated in only one sample (if u does not saturate!). Open the valve to verify this behavior. Also, the reference is tracked in one sample if the step change is small enough not to cause the control signal to saturate.

- Make a large setpoint change, for example try to fill up to a level very close to the top. Also, try to empty from a full tank to a level close to the bottom. The limitation of the control signal causes integrator windup.

- Click in the first window to choose anti-windup and repeat the experiment above. What is the difference in performance?