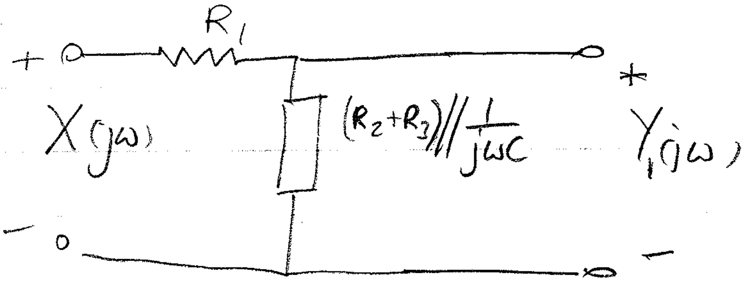
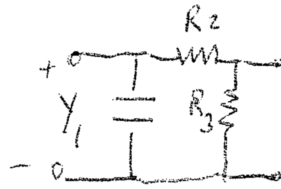
LÖSNINGSFÖRSLAG TILL TENTAMEN 050110 I SIGNALER OCH SYSTEM för E2/D2/Mek2

1.



$$Y_1(j\omega) = X(j\omega)\cdot\frac{Z(j\omega)}{Z(j\omega)+R_1}$$

$$Z(j\omega) = \frac{(R_2+R_3)\cdot\frac{1}{j\omega C}}{R_2+R_3+\frac{1}{j\omega C}} = \frac{R_2+R_3}{j\omega C(R_2+R_3)+1}$$



$$Y(j\omega) = Y_1\cdot\frac{R_3}{R_2+R_3}$$

$$Y_1(j\omega) = X(j\omega)\cdot\frac{\frac{R_2+R_3}{j\omega C(R_2+R_3)+1}}{\frac{R_2+R_3}{j\omega C(R_2+R_3)+1}+R_1} = X(j\omega)\cdot\frac{R_2+R_3}{(R_2+R_3)+R_1\left(1+j\omega C(R_2+R_3)\right)} \Rightarrow$$

$$Y(j\omega) = X(j\omega)\cdot\frac{R_3}{(R_2+R_3)+R_1\left(1+j\omega C(R_2+R_3)\right)}$$

$$\Rightarrow \frac{Y(j\omega)}{X(j\omega)} = \frac{3M}{5M+1M\left(1+j\omega\, 1\cdot 10^{-6}\, 5\cdot 10^{6}\right)} = \frac{3M}{5M+1M(1+5\omega j)} = \frac{3}{6+5\omega j}$$

$$|H(\omega)| = \frac{3}{\sqrt{36+25\omega^2}} \qquad \arg H(\omega) = -\arctan\left(\frac{5\omega}{6}\right) \quad [\text{rad}]$$

$$|H(1)| = \frac{3}{\sqrt{61}} \qquad |H(3)| = \frac{3}{\sqrt{36+225}} = \frac{3}{\sqrt{261}}$$

$$\arg H(1) = -\arctan\left(\frac{5}{6}\right) \qquad \arg H(3) = -\arctan\left(\frac{15}{6}\right)$$

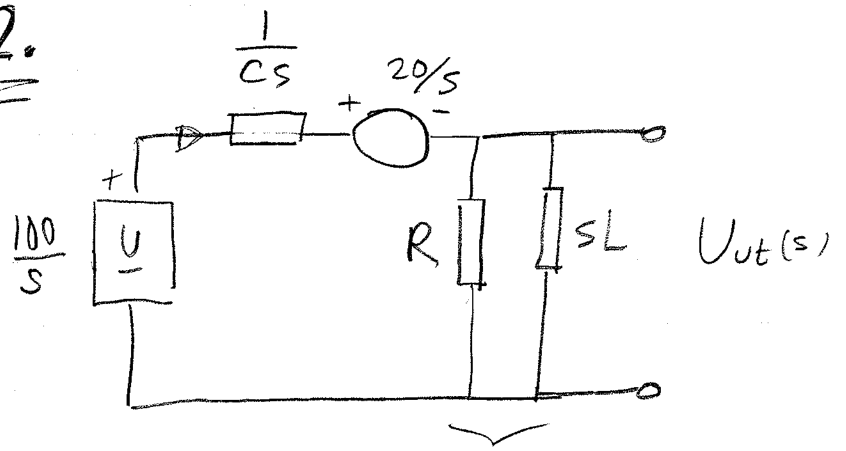
$$y(t) = \frac{3}{\sqrt{61}}\cdot\sin\left(t-\arctan\left(\frac{5}{6}\right)\right) + \frac{3}{\sqrt{261}}\cdot\sin\left(3t-\arctan\frac{15}{6}\right)$$

$$P_{R_3} = \frac{U_{3e}^2}{R_3} \Rightarrow U_{3e} = \sqrt{\left(\frac{3}{\sqrt{122}}\right)^2+\left(\frac{3}{\sqrt{522}}\right)^2} = 3\sqrt{\frac{1}{122}+\frac{1}{522}} \simeq 0{,}302$$

$$P_{R_3} = 30{,}4 \text{ nW}$$

2.

Bestäm för $t > 0$!



$$\underbrace{}_{\frac{R\cdot sL}{R+sL}}$$

$$U_{ut}(s) = \left(\frac{100}{s} - \frac{20}{s}\right) \cdot \frac{\frac{R\cdot sL}{R+sL}}{\frac{R\cdot sL}{R+sL} + \frac{1}{Cs}} = \frac{80}{s} \cdot \frac{sLR}{sLR + \frac{(R+sL)}{Cs}} = \frac{80}{s} \cdot \frac{s^2LCR}{s^2LCR + R + sL}$$

$$= \left\{ \begin{matrix} R = 1000 \\ L = 0{,}1 \\ C = 2\cdot 10^{-7} \end{matrix} \right\} = 80 \cdot \frac{s \cdot 2\cdot 10^{-5}}{s^2 \cdot 2 \cdot 10^{-5} + 1000 + s \cdot 0{,}1} = \frac{80 \cdot s}{s^2 + \frac{0{,}1\, s}{2\cdot 10^{-5}} + \frac{1000}{2\cdot 10^{-5}}}$$

$$= \frac{80 \cdot s}{s^2 + 5000s + 50 \cdot 10^6} = \frac{80(s+2500) - 2500\cdot 80}{(s+2500)^2 + 43{,}75 \cdot 10^6} \approx \frac{80(s+2500) - 2500\cdot 80}{(s+2500)^2 + 6614{,}4^2}$$

$$U_{ut}(t) \approx 80 e^{-2500t} \cdot \cos(6614{,}4t) - 30{,}2 \cdot e^{-2500t} \sin(6614{,}4t)$$

3.

$$s^2 Y(s) + 3s Y(s) + 2\, Y(s) = X(s)$$

Antag att B.V. $= 0$ (jämvikt råder)

$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2+3s+2} = \frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2}$$

a) $g(t) = (e^{-t} - e^{-2t})u(t)$

b) Stabilt alla poler ligger i VHP.

c) $Y(s) = \underbrace{X(s)}_{\text{insignal är ett steg}} \cdot \underbrace{G(s)}$ $\quad Y(0) = X(0) \cdot G(0) = \frac{1}{2}$

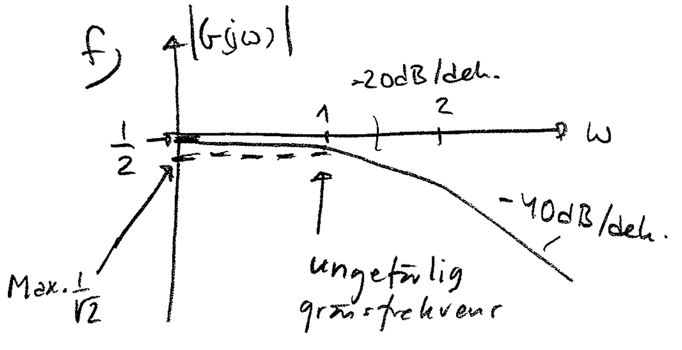
eller $Y(\infty) = \lim_{s \to 0} s \cdot \underbrace{X(s)}_{\frac{1}{s}} \cdot G(s) = \lim G(0) = \frac{1}{2}$

d) $|G(j\omega)| = \dfrac{1}{\sqrt{\omega^2+1}\,\sqrt{\omega^2+4}}$ $\quad \arg G(j\omega) = -\arctan\left(\frac{\omega}{1}\right) - \arctan\left(\frac{\omega}{2}\right)$

amplitudkarakteristiken $\quad$ faskarakteristiken

e) $G(s) = \dfrac{1}{s^2+3s+2}$

f)



Systemet är ett LP-filter och har i princip endast 1 övre gränsfrekvens

Beräknas $|G(j\omega_{gö})| = \dfrac{1}{\sqrt{\omega^2+1}\,\sqrt{\omega^2+4}} = \dfrac{1}{2} \cdot \dfrac{1}{\sqrt{2}}$ $\rightarrow \omega_{gö} \approx 0{,}83$

4. Bestäm systemfunktionen för ett LP-filter med max flat amplitudkurva

$\Downarrow$

a) Butterworthfilter

	dämpning		frekvens
	$A_p = 3\,dB$	$\rightarrow$	$4\,kHz$ (f_0)
	$A_s = 40\,dB$	$\rightarrow$	$40\,kHz$ (f_s)

$$n \geq \frac{{}^{10}\log\left(\frac{10^{0,1A_s}-1}{10^{0,1A_p}-1}\right)}{2\cdot{}^{10}\log\frac{\omega_s}{\omega_0}} = 2$$

$$H(s) = \frac{1}{1 + 1{,}4142\cdot\left(\frac{s}{\Omega_0}\right) + 1\cdot\left(\frac{s}{\Omega_0}\right)^2}$$

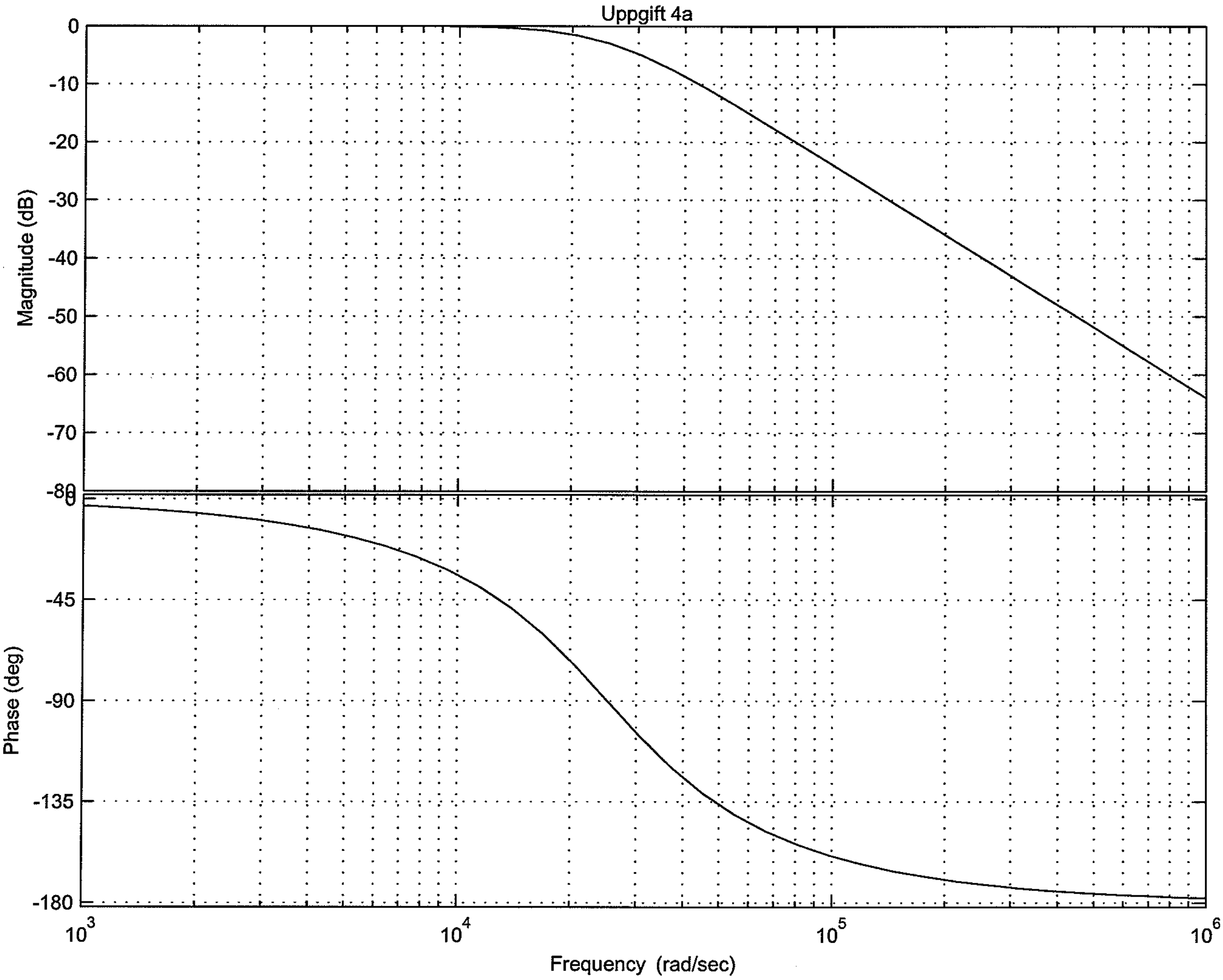
$$\boxed{\begin{aligned}\Omega_0 &= 2\pi f_0\\ &\approx 25133\end{aligned}}$$

b) Konstruera ett HP-filter Butterworth från ovanstående LP-filter, med gränsfrekvens är 1kHz. ($\omega_0 = 1000\cdot 2\pi$).

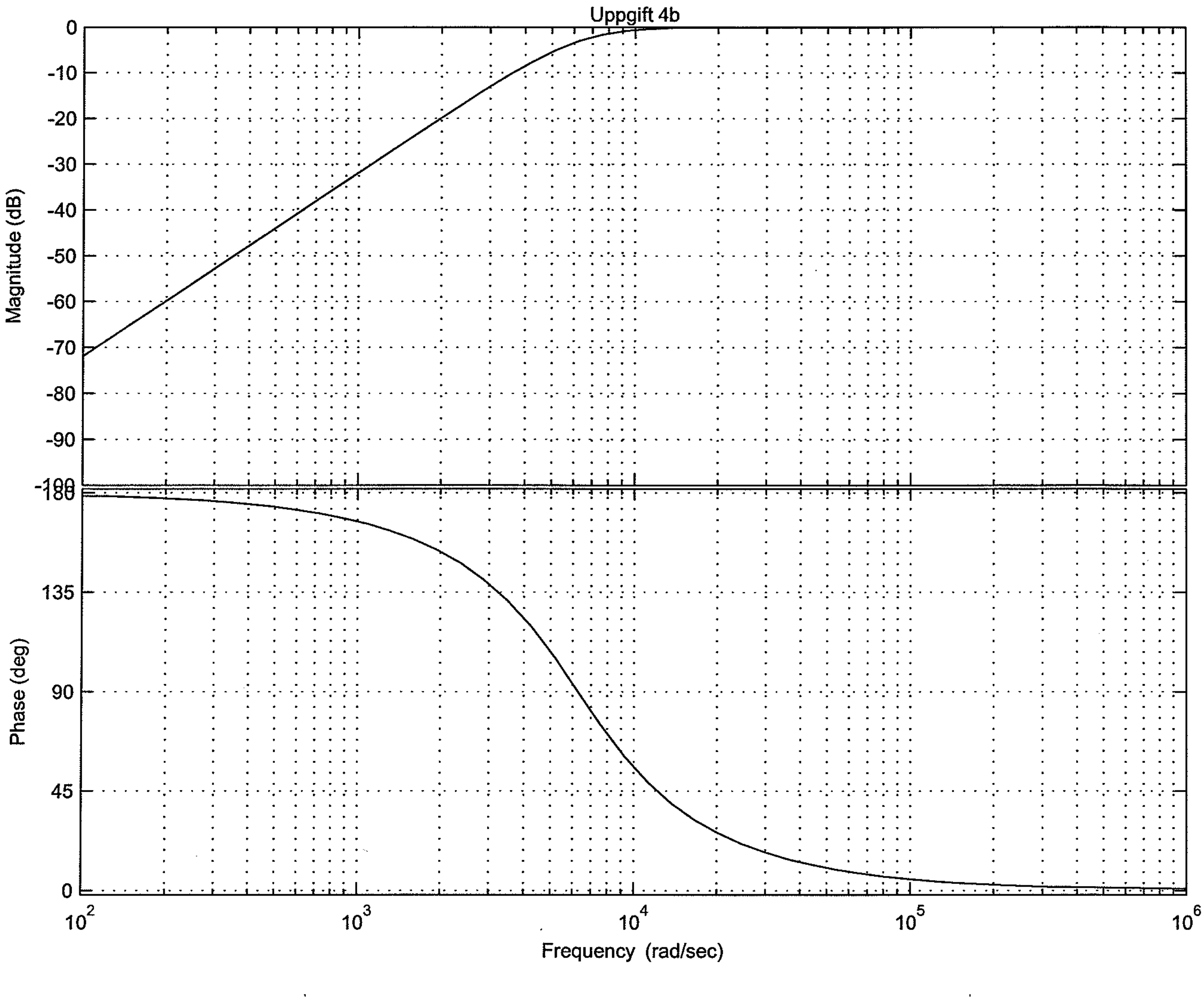
$\omega_I^2 = \Omega_0\cdot\omega_0$ enligt tabell 3 $\quad$ LP$\rightarrow$HP $\quad s \rightarrow \frac{\omega_I^2}{s} = \frac{\Omega_0\omega_0}{s}$

$$H_{HP}(s) = \frac{1}{1 + 1{,}4142\cdot\left(\frac{\cancel{\Omega_0}\cdot\omega_0}{s\cdot\cancel{\Omega_0}}\right) + \left(\frac{\cancel{\Omega_0}\cdot\omega_0}{s\cdot\cancel{\Omega_0}}\right)^2} =$$

$$= \frac{s^2}{s^2 + 1{,}4142\,\omega_0 s + \omega_0^2} = \left\{\omega_0 \approx 6283\right\} =$$



Uppgift 4a
Magnitude (dB)
0
-10
-20
-30
-40
-50
-60
-70
-80
Phase (deg)
0
-45
-90
-135
-180
10^3
10^4
10^5
10^6
Frequency (rad/sec)


Uppgift 4b
Magnitude (dB)
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
Phase (deg)
180
135
90
45
0
10^2
10^3
10^4
10^5
10^6
Frequency (rad/sec)

5. $x(t) = e^{-2t} \cdot u(t)$ fungerar som insignal till idealt LP-filter.

$$H(\omega) = \begin{cases} 1 & , -2\pi W \le \omega \le 2\pi W \\ 0 & , \text{f.ö.} \end{cases}$$

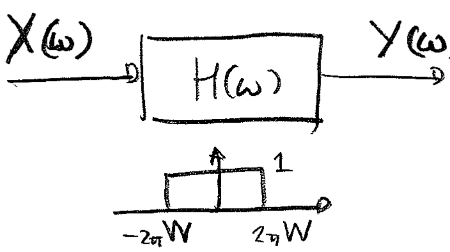
För vilket värde på filtrets bandbredd W kommer hälften av signalens energi igenom LP-filtret.

$$X(\omega) = \frac{1}{j\omega + 2} \qquad |X(\omega)| = \frac{1}{\sqrt{\omega^2+4}}$$

energiinnehåll : $W = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{\omega^2+4}\,d\omega = \frac{1}{\pi}\int_{0}^{\infty} \frac{d\omega}{\omega^2+4} =$

$$= \frac{1}{\pi 4}\int_0^{\infty} \frac{d\omega}{\left(\frac{\omega}{2}\right)^2+1} = \left\{ \begin{array}{l} \frac{\omega}{2} = \omega' \\ \frac{1}{2}d\omega = d\omega' \end{array} \right\} = \frac{1}{4\pi}\int_0^{\infty} \frac{2\,d\omega'}{\omega'^2+1} = \frac{1}{2\pi}\Big[\arctan \omega'\Big]_0^{\infty}$$

$= \frac{1}{2\pi}\left[\frac{\pi}{2} - 0\right] = \frac{1}{4}$ energiinnehållet är $\frac{1}{4}$ över hela frekv. området.



$Y(\omega) = X(\omega) \cdot H(\omega) = X(\omega)$

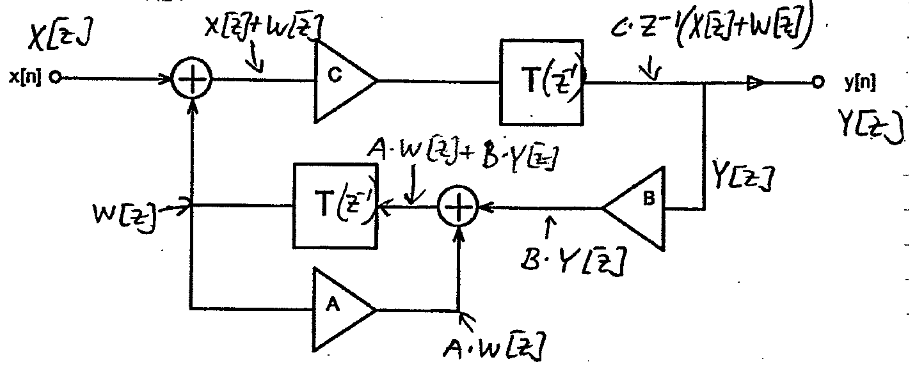
$$W = \frac{1}{2\pi}\int_{-2\pi W}^{2\pi W} \frac{1}{\omega^2+4} = \ldots = \frac{1}{2\pi}\Big[\arctan \omega'\Big]_0^{2\pi W} = \frac{1}{8}$$

$$2\pi W = \tan\frac{\pi}{4} \implies W = \frac{1}{2\pi}\tan\frac{\pi}{4} = \frac{1}{2\pi}$$

SVAR: BANDBREDD ÄR $\frac{1}{2\pi}$ Hz

Lösningar till tenta Signaler & System, tidsdiskret del 10/1-05

1b.



1)
$$W[z] = z^{-1}\left(A\cdot W[z] + B\cdot Y[z]\right)$$

$$\Rightarrow W[z]\left(1 - A\cdot z^{-1}\right) = z^{-1}\cdot B\cdot Y[z]$$

$$\Rightarrow W[z] = \frac{B\cdot z^{-1}}{1-A\cdot z^{-1}}\cdot Y[z] = \frac{B}{z-A}\cdot Y[z]$$

2)
$$Y[z] = C\cdot z^{-1}\cdot\left(X[z] + W[z]\right) = C\cdot z^{-1}\cdot X[z] +$$
$$+ \frac{C\cdot z^{-1}\cdot B}{z-A}\cdot Y[z]$$

$$\Rightarrow Y[z]\left(1 - \frac{B\cdot C\cdot z^{-1}}{z-A}\right) = C\cdot z^{-1}\cdot X[z]$$

$$\Rightarrow H[z] = \frac{Y[z]}{X[z]} = \frac{C\cdot z^{-1}}{1 - \frac{B\cdot C\cdot z^{-1}}{z-A}}\cdot\frac{(z-A)\cdot z}{(z-A)\cdot z}$$

$$= \frac{C(z-A)}{z(z-A) - B\cdot C} = \frac{0{,}5(z-1)}{z(z-1) - (-0{,}5)\cdot 0{,}5} =$$

$$= \frac{0{,}5(z-1)}{z^2 - z + 0{,}25} = \frac{C_1}{z-0{,}5} + \frac{C_2\cdot z}{(z-0{,}5)^2}$$

$$\therefore H[z] = \frac{C_1(z-0{,}5) + C_2\cdot z}{(z-0{,}5)^2} = \frac{0{,}5(z-1)}{(z-0{,}5)^2}$$

$$C_1 + C_2 = 1 \quad \text{och} \quad -C_1\cdot 0{,}5 = -0{,}5 \Rightarrow C_1 = 1 \text{ och } C_2 = -0{,}5$$

$$\Rightarrow H[z] = z^{-1}\,\frac{z}{z-0{,}5} - \frac{0{,}5\cdot z}{(z-0{,}5)^2}$$

Formelsamling $\Rightarrow$ Impulssvar $\underline{h[n] = 0{,}5^{n-1}\cdot u[n-1] - n\cdot 0{,}5^{n}\cdot u[n]}$

7. Polerna och nollställena ger

$$H[z] = K \cdot \frac{(z-1)(z+j)(z-j)}{(z+0,5)(z+0,75+j0,25)(z+0,75-j0,25)}$$

$$= K \cdot \frac{(z-1)(z^2+1)}{(z+0,5)(z^2+1,5\cdot z+0,625)}$$

Polerna inom enhetscirkeln $z = e^{j\Omega} \Rightarrow$ stabilt system

Frekvensfunktionen $H[\Omega] = K \cdot \frac{(e^{j\Omega}-1)(e^{j2\Omega}+1)}{(e^{j\Omega}+0,5)(e^{j2\Omega}+1,5\cdot e^{j\Omega}+0,625)}$

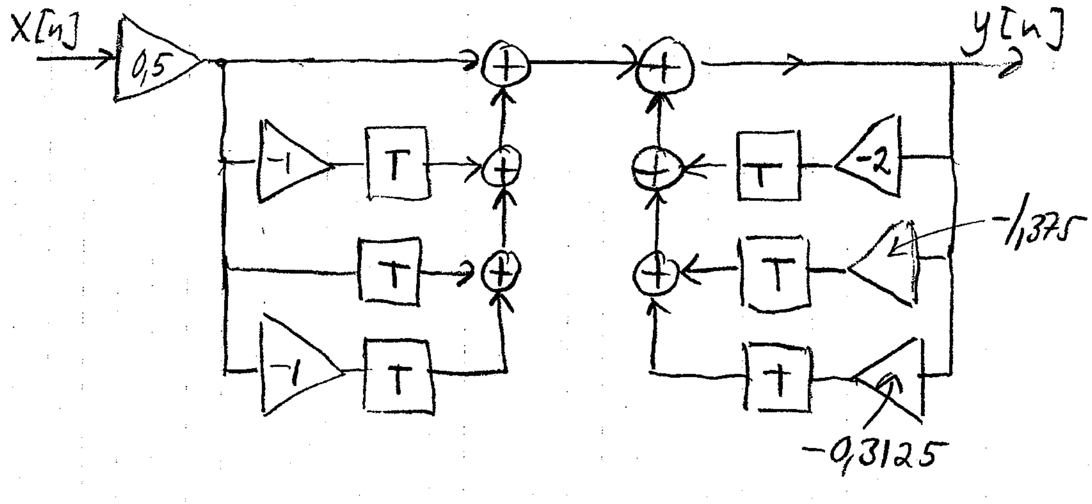
$$\Rightarrow H[\pi] = 32 = K \cdot \frac{(-1-1)(1+1)}{(-1+0,5)(1-1,5+0,625)} = \frac{-4}{-0,0625}$$

$$\Rightarrow K = 0,5$$

$$\therefore H[z] = 0,5 \cdot \frac{z^3-z^2+z-1}{z^3+2\cdot z^2+1,375\cdot z+0,3125} \cdot \frac{z^{-3}}{z^{-3}} = \frac{Y[z]}{X[z]}$$

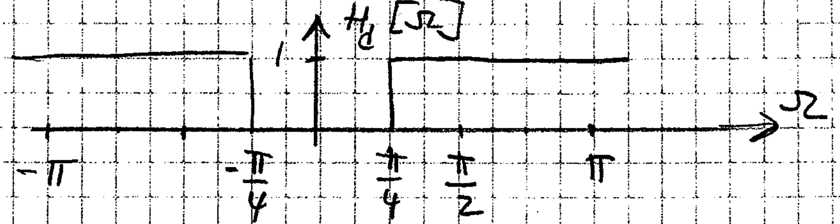
$$\Rightarrow Y[z]\left(1+2\cdot z^{-1}+1,375\cdot z^{-2}+0,3125\cdot z^{-3}\right) = 0,5\cdot X[z]\left(1-z^{-1}+\right.$$
$$\left.+z^{-2}-z^{-3}\right)$$

$$\Rightarrow y[n]+2\cdot y[n-1]+1,375\cdot y[n-2]+0,3125\cdot y[n-3] =$$
$$= 0,5\cdot\left(x[n]-x[n-1]+x[n-2]-x[n-3]\right)$$



8/ Normerad vinkelfrekvens $\Omega_0 = \omega_0 T = \frac{2\pi f_0}{f_s} =$

$$= \frac{2\pi \cdot 2{,}5\ kHz}{20\ kHz} = \frac{\pi}{4}$$



HP-filtrets impulssvar

$$h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{-\frac{\pi}{4}} H_d[\Omega] \cdot d\Omega +$$

$$+ \frac{1}{2\pi} \int_{\pi/4}^{\pi} H_d[\Omega] \cdot d\Omega = \frac{1}{2\pi} \left\{ \left[\frac{e^{j\Omega n}}{jn} \right]_{-\pi}^{-\frac{\pi}{4}} + \left[\frac{e^{j\Omega n}}{jn} \right]_{\pi/4}^{\pi} \right\}$$

$$= \frac{1}{2\pi j n} \left(e^{j\pi n} - e^{-j\pi n} + e^{-j\frac{\pi}{4} \cdot n} - e^{j\frac{\pi}{4} \cdot n} \right) =$$

$$= \frac{1}{2\pi j n} \left(2j \underbrace{\sin \pi n}_{=0} - 2j \sin \frac{\pi}{4} n \right) = -\frac{1}{\pi n} \cdot \sin \frac{\pi}{4} \cdot n$$

Eulers formler

då $n \neq 0$

$$n = 0 \Rightarrow h_d[0] = \frac{1}{2\pi} \int_{\pi}^{-\frac{\pi}{4}} d\Omega + \frac{1}{2\pi} \int_{\pi/4}^{\pi} d\Omega =$$

$$= \frac{1}{2\pi} \left(-\frac{\pi}{4} + \pi + \pi - \frac{\pi}{4} \right) = \frac{3}{4}$$

$$h_d[1] = -\frac{1}{\pi} \sin \frac{\pi}{4} = -0{,}225 = h_d[-1]$$

$$h_d[2] = -\frac{1}{2\pi} \sin \frac{\pi}{2} = -0{,}159 = h_d[-2]$$

$$h_d[3] = -\frac{1}{3\pi} \sin \frac{3\pi}{4} = -0{,}075 = h_d[-3]$$

Skifta 3 steg och multiplicera med Barlett fönstrets viktfunktion: $w[0] = 0$, $w[1] = \frac{1}{3}$, $w[2] = \frac{2}{3}$, $w[3] = 1$, $w[4] = \frac{2}{3}$, $w(5) = \frac{1}{3}$, $w[6] = 0$

$$\begin{cases} C_0 = -0{,}075 \cdot 0 = \underline{0} = C_6 \\ C_1 = -0{,}159 \cdot \frac{1}{3} = \underline{-0{,}053} = C_5 \\ C_2 = -0{,}225 \cdot \frac{2}{3} = \underline{-0{,}15} = C_4 \\ C_3 = 0{,}75 \cdot 1 = \underline{0{,}75} \end{cases}$$

FIR-filterrealisering
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