PREL.

LÖSNINGSFÖRSLAG TILL TENTAMEN 050323 i Signaler o System för D2/E2/Mek2

1

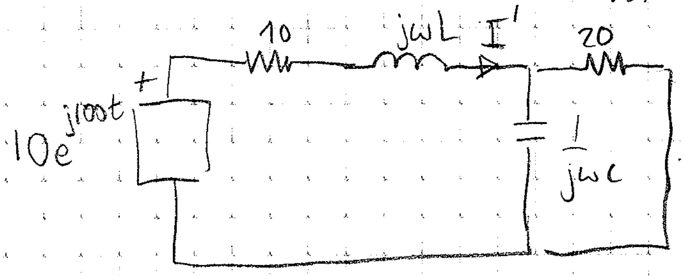
$$\overline{u} = \frac{1\cdot 0{,}5}{4} = 0{,}25 \text{ Volt}$$

$$|\overline{u}| = 0{,}25 \text{ Volt}$$

$$u_{RMS} = \sqrt{\frac{1}{2}\int_0^{0,5} 1^2\,dt} = \sqrt{\frac{1}{2}\Big[t\Big]_0^{0,5}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

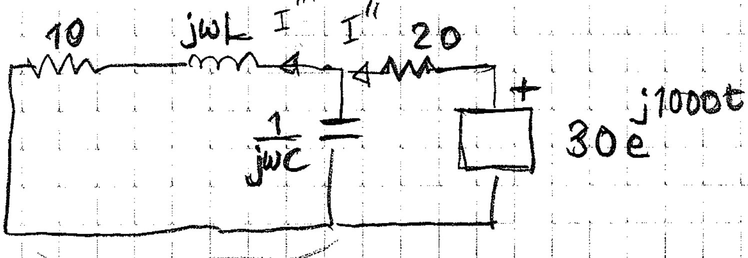
2. Bestäm m h a superposition!

Först tittar vi på $e_1(t)$ och ersätt $e_2(t)$ med kortslutning.



$$z_p = \frac{20\cdot\frac{1}{j\omega C}}{20+\frac{1}{j\omega C}} = \frac{20}{j\omega C\cdot 20+1}$$

$$I' = \frac{10e^{j100t}}{10+j\omega L+\frac{20}{j\omega C20+1}}$$



$$I'' = \frac{30e^{j1000t}}{20+\frac{10+j\omega L}{j\omega 10C-\omega^2 LC+1}}$$

$$Z_p = \frac{(10+jL\omega)\cdot\frac{1}{j\omega C}}{10+j\omega L+1/j\omega C} = \frac{10+j\omega L}{j\omega 10C-\omega^2 LC+1}$$

2 forts. Totala strömmen blir : $I = I' + I''$

$$I' = \frac{10\,(j\omega C 20 + 1)}{(10 + j\omega L)(j\omega C 20 + 1) + 20} = \frac{j\omega C 200 + 10}{10 - \omega^2 LC 20 + j\omega L + j\omega C 200 + 20}$$

$$\Rightarrow |I'| = \frac{\sqrt{0{,}2^2 + 10^2}}{\sqrt{(30 - 0{,}002)^2 + (0{,}1 + 0{,}2)^2}} \simeq \frac{10}{30} = \frac{1}{3}$$

$$\arg I' \approx 0^\circ$$

$$I'' = \frac{30}{20 + \dfrac{10 + j\omega L}{j\omega 10C - \omega^2 LC + 1}} = \frac{30\,(j\omega 10C - \omega^2 LC + 1)}{20\,(j\omega 10C - \omega^2 LC + 1) + 10 + j\omega L}$$

$$\arg I'' \approx 0^\circ$$

$$|I''| = \frac{30 \cdot \sqrt{(1 - 100 \cdot 1 \cdot 10^{-8})^2 + 0{,}01^2}}{\sqrt{(30 - 20 \cdot 100^2 \cdot 10^{-8})^2 + (0{,}1 + 0{,}2)^2}} \simeq \frac{30}{30} = 1$$

strömdela!

$$I''' = I'' \cdot \frac{\frac{1}{j\omega C}}{10 + j\omega L + \frac{1}{j\omega C}} = \frac{I'' \cdot 1}{j\omega C \cdot 10 - \omega^2 LC + 1} \Rightarrow |I'''| = I'' \frac{1}{\sqrt{0{,}1^2 + 0{,}99^2}} \simeq 1$$

$$\arg I''' = 0^\circ$$

$$i(t) \simeq \frac{1}{3} \sin 100\,t \; - 1 \sin(1000\,t)$$

Effektivvärde:

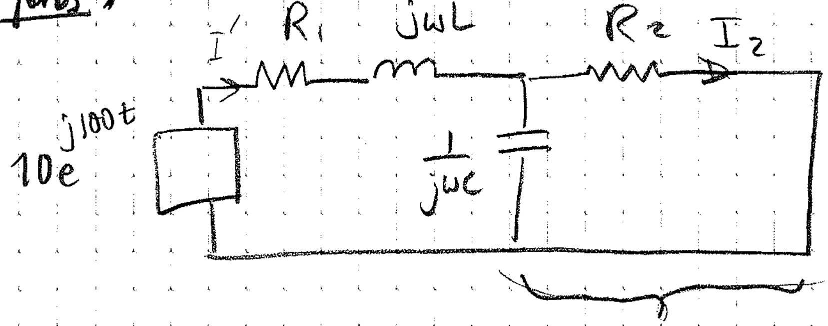
$$I^2 = \left(\frac{I'}{\sqrt{2}}\right)^2 + \left(\frac{I'''}{\sqrt{2}}\right)^2 \Rightarrow I = \sqrt{\left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{18} + \frac{1}{2}} = \frac{\sqrt{5}}{3}$$

b) Aktiv effekt som utvecklas i R_1 blir :

$$P_{R_1} = R_1 \left(\frac{I'}{\sqrt{2}}\right)^2 + R_1 \left(\frac{I'''}{\sqrt{2}}\right)^2 = 10\left(\frac{1}{18} + \frac{9}{18}\right) = \frac{100}{18}\,[W] = \frac{50}{9}$$

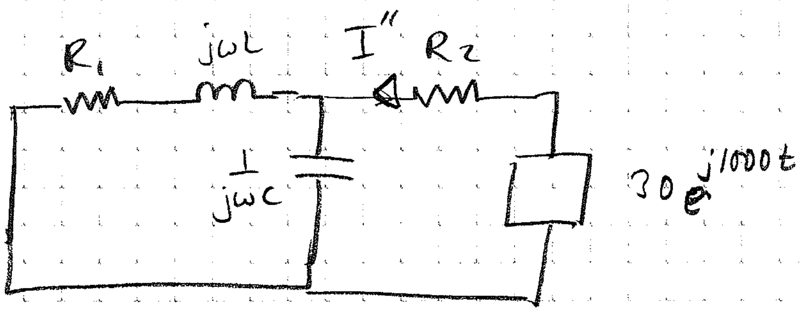
För att bestämma aktiva effekten i R_2 måste strömmen bestämmas. Använd på nytt superposition.

2b forts:



$$I_2 = I' \cdot \frac{\frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = I' \cdot \frac{1}{j\omega C R_2 + 1}$$

$$|I_2| = |I'| \cdot \frac{1}{\sqrt{(100 \cdot 10^{-5} \cdot 20)^2 + 1}} \approx |I'|$$



$$P_{R_2} = R_2\left(\frac{I''}{\sqrt{2}}\right)^2 + R_2\left(\frac{I_2}{\sqrt{2}}\right)^2 = 20\left(\frac{1}{2} + \frac{1}{18}\right) = 20 \cdot \frac{10}{18} = \frac{100}{9} \, \underline{[W]}$$

3.

$$H_{tot}(s) = \frac{\cancel{s+1}}{10s+1} \cdot \frac{(-s+1)}{(s+1)^{\cancel{2}}} \cdot \frac{1}{s+10}$$

$$H_{tot}(\omega) = \frac{1}{10j\omega+1} \cdot \frac{(-j\omega+1)}{(j\omega+1)} \cdot \frac{1}{(j\omega+10)}$$

Amplitudkarakteristik: $|H_{tot}(\omega)| = \frac{1}{\sqrt{100\omega^2+1}} \cdot \frac{\cancel{\sqrt{\omega^2+1}}}{\cancel{\sqrt{\omega^2+1}}} \cdot \frac{1}{\sqrt{(\omega^2+100)}}$

$$= \frac{1}{\sqrt{100\omega^2+1}} \cdot \frac{1}{\sqrt{\omega^2+100}}$$

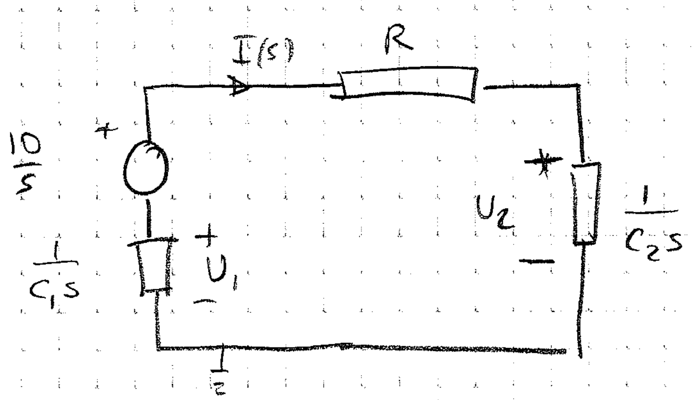
Faskarakteristiken: $\arg H_{tot}(\omega) = -\arctan\omega -$

$$- \arctan(10\omega) - \arctan\omega - \arctan\frac{\omega}{10}$$

Se bifogat Bodediagram!

Systemet $H_{tot}(s)$ är stabilt eftersom samtliga poler ligger i VHP. $s = -\frac{1}{10}, -10, -1, -1$

4.



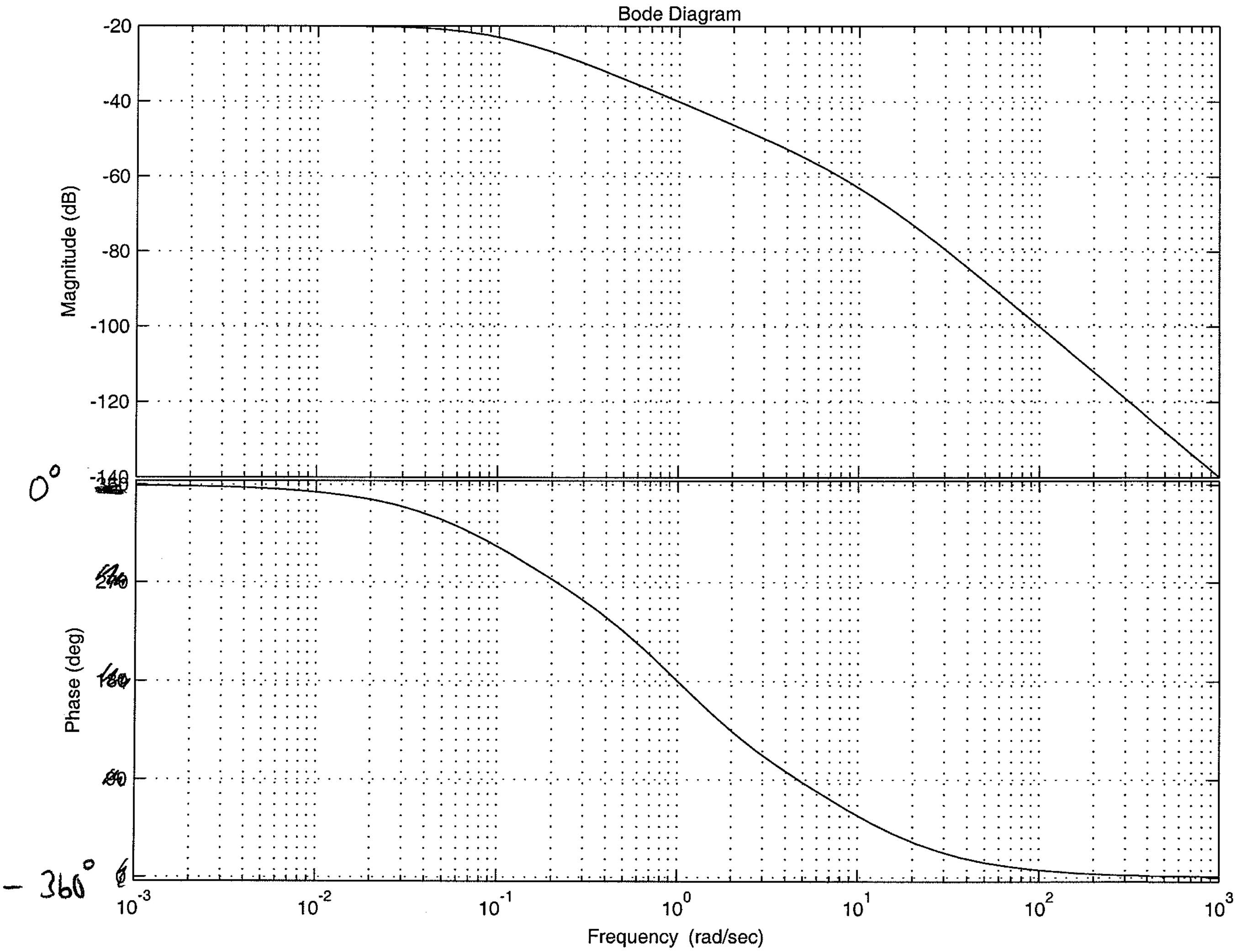
$$\begin{cases} C_1 = 1\mu F \\ C_2 = 0.2\mu F \\ R = 10k\Omega \end{cases}$$

$$I(s) = \frac{10/s}{\frac{1}{C_1 s} + \frac{1}{C_2 s} + R} = \frac{10}{\frac{1}{C_1} + \frac{1}{C_2} + Rs} = \frac{10}{10^6 + 5\cdot 10^6 + 10^4 s} = \frac{10}{6\cdot 10^6 + 10^4 s}$$

$$U_2(s) = \frac{1}{C_2 s} \cdot I(s) = \frac{5\cdot 10^6}{s} \cdot \frac{10}{6\cdot 10^6 + 10^4 s} = \frac{50}{6s + 0{,}01 s^2} = \frac{50/6}{s + 1{,}667\cdot 10^{-3} s^2} =$$

$$\frac{A}{s} + \frac{B}{s\,1{,}667\cdot 10^{-3} + 1} = \frac{A\cdot 1{,}667\cdot 10^{-3} s + A + Bs}{s\,(s\,1{,}667\cdot 10^{-3} + 1)} \Rightarrow \quad A \approx 8{,}33 \quad B = -8{,}33$$

3.


Bode Diagram
Magnitude (dB)
-20
-40
-60
-80
-100
-120
-140
Phase (deg)
0°
-360°
Frequency (rad/sec)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

4 forts

$$u_2(t) = 8{,}33\left(1 - e^{-600t}\right)$$

$$U_1(s) = \frac{1}{C_1 s}\cdot I(s) = \frac{10^6}{s}\cdot\frac{10}{6\cdot10^6 + 10^4 s} = \frac{10}{s(6+0{,}01s)} = \frac{10/6}{s(1+1{,}667\cdot10^{-3}s)}$$

$$= \frac{C}{s} + \frac{D}{(s\,1{,}667\cdot10^{-3}+1)} = \frac{C(s\,1{,}667\cdot10^{-3}+1) + Ds}{s(s\,1{,}667\cdot10^{-3}+1)} \approx$$

$$\approx \begin{cases} C = 1{,}67 \\ D = -1{,}67 \end{cases} \qquad u_1(t) = 1{,}67\left(1 - e^{-t600}\right)$$

med hänsyn taget till B.V.

$$u_1(t) = \underbrace{10 - 1{,}67}_{} + 1{,}67\cdot e^{-t600} = 8{,}33 + 1{,}67\cdot e^{-t600}$$

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5. Konstruera ett Chebyshev LP-filter med $\omega_0 = 2000\,\text{rad/s}$ och 1dB rippel.

a) Detta LP-filter skall ha en dämpning på minst 40dB vid 20krad/s.

Avläsning ur tabell 7b. Chebyshev + 1dB rippel.
Vi vet att ordningtalet är 2 eftersom kurvan dämpas -40dB på en dekad.

$$H_{LP}(s) = \frac{1}{\left(\frac{s}{2000} + 0{,}5489 - j\,0{,}8951\right)\left(\frac{s}{2000} + 0{,}5489 + j\,0{,}8951\right)}$$

ω_{0LP}

b) Konstruera ett HP-filter utgående ifrån ovanstående!

LP → HP

$s \to \frac{\omega_I^2}{s}$

$$\omega_I^2 = \omega_{0LP}\cdot\omega_{0HP} = 2000\cdot1643$$

För gradtal $n=2$ & 1dB rippel fås m h a tabell 8

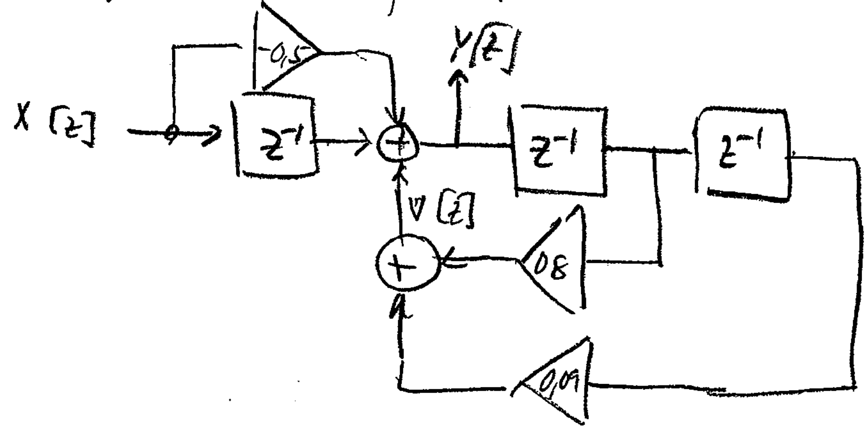
$$\frac{\omega_{3dB}}{\omega_{rippel}} = 1{,}2176$$

$$\omega_{rippel} = \omega_{0HP} = \frac{2000}{1{,}2176} = 1643\ \text{rad/s}$$

$$H_{HP}(s) = \frac{1}{\left(\frac{1643}{s} + 0{,}5489 - j\,0{,}8951\right)\left(\frac{1643}{s} + 0{,}5489 + j\,0{,}8951\right)}$$

ω_{0HP}

Ex 6. Övergå till z-planet



$$\begin{cases} Y[z] = X[z]\cdot z^{-1} + V[z] - X[z]\cdot 0{,}5 \\ Y[z] = Y[z]\cdot z^{-1}\cdot 0{,}8 + Y[z]\cdot z^{-2}\cdot 0{,}09 \end{cases}$$

$$\Rightarrow \quad Y[z] = X[z]\left(z^{-1}-0{,}5\right) + Y[z]\cdot\left(0{,}8\cdot z^{-1}+0{,}09z^{-2}\right)$$

$$\Rightarrow \quad Y[z]\left(1-0{,}8z^{-1}-0{,}09z^{-2}\right) = X[z]\left(z^{-1}-0{,}5\right)$$

$$\Rightarrow \quad H[z] = \frac{Y[z]}{X[z]} = \frac{z^{-1}-0{,}5}{1-0{,}8z^{-1}-0{,}09\cdot z^{-2}} =$$

$$= \frac{z(1-0{,}5z)}{z^2-0{,}8z-0{,}09} = \frac{z(1-0{,}5z)}{(z-0{,}9)(z+0{,}1)}$$

$$= z\left(\frac{A}{z-0{,}9}+\frac{B}{z+0{,}1}\right) = z\,\frac{A(z+0{,}1)+B(z-0{,}9)}{(z-0{,}9)(z+0{,}1)}$$

$$\therefore \begin{cases} A+B=-0{,}5 \\ A\cdot 0{,}1 - B\cdot 0{,}9 = 1 \Rightarrow A = 10+9B \end{cases}$$

$$10+9B+B=-0{,}5$$
$$10B=-10{,}5$$
$$B=\underline{-1{,}05}$$
$$A=1{,}05-0{,}5=$$
$$=\underline{0{,}55}$$

$$= z\left(\frac{0{,}55}{z-0{,}9}+\frac{-1{,}05}{z+0{,}1}\right)$$

$$\Rightarrow h[n] = \left[0{,}55\cdot 0{,}9^n - 1{,}05\cdot(-0{,}1)^n\right]\cdot u[n]$$

7.

$$H[z] = k\frac{(z+1)(z-1)}{(z+j0{,}9)(z-j0{,}9)} = k\cdot\frac{z^2-1}{z^2+0{,}81} =$$

$$= k\frac{1-z^{-2}}{1+0{,}81\cdot z^{-2}} = \frac{Y[z]}{X[z]}$$

$$\Rightarrow X[z] - X[z]\cdot z^{-2} = Y[z] + Y[z]\cdot 0{,}81\cdot z^{-2}$$

$$\underline{\underline{k\cdot x[n] - k x[n-2] = y[n] + 0{,}81\, y[n-2]}}$$

Stabilt system $z = e^{j\Omega}$

$$H[\Omega] = k\,\frac{e^{j2\Omega}-1}{e^{j2\Omega}+0{,}81} = k\cdot\frac{\cos 2\Omega + j\sin 2\Omega - 1}{\cos 2\Omega + j\sin 2\Omega + 0{,}81} =$$

$$= |H[\Omega]|^2 = k^2\frac{(\cos 2\Omega - 1)^2 + \sin^2 2\Omega}{(\cos 2\Omega + 0{,}81)^2 + \sin^2 2\Omega} =$$

$$= k^2\frac{\cos^2 2\Omega - 2\cos 2\Omega + 1 + \sin^2 2\Omega}{\cos^2 2\Omega + 2\cdot 0{,}81\cos 2\Omega + 0{,}81^2 + \sin^2 2\Omega} =$$

$=1$

$$\underline{\underline{= k^2\frac{2-2\cos 2\Omega}{1+0{,}81^2 + 1{,}62\cos 2\Omega}}}$$

$$H[\pi/2] = 20 \Rightarrow 20 = k\frac{e^{j\pi}-1}{e^{j\pi}+0{,}81} = k\frac{-1-1}{-1+0{,}81} \Rightarrow k = 1{,}9$$

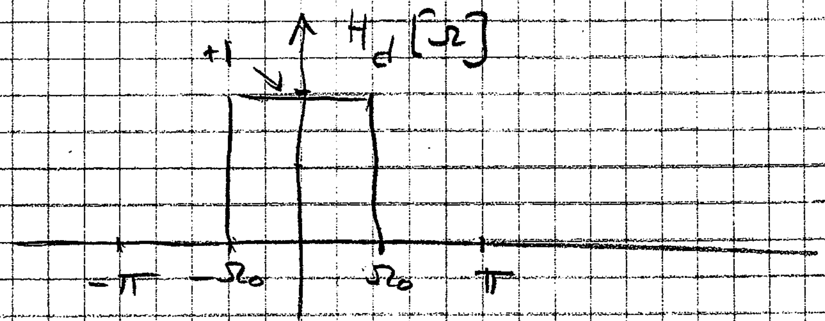
$|H[\Omega]|_{max}$ då $\cos 2\Omega = -1 \Rightarrow 2\Omega = \pi \Rightarrow \Omega = \frac{\pi}{2}$

$$\Omega = \omega T = \frac{\pi}{2} = \omega\cdot 10^{-3} \Rightarrow \omega = 1570 \text{ rad/s}$$

$$\Rightarrow f = \frac{\omega}{2\pi} = \frac{\pi\cdot 10^3}{2\cdot 2\pi} = \underline{\underline{250\ Hz}}$$ (mitt frekvens)

8.

$$\Omega_0 = \omega_0 T = \frac{2\pi f_0}{f_s} = \frac{2\pi \cdot 5}{44{,}1} = \frac{10\pi}{44{,}1} = 0{,}227\,\pi$$



Impulssvar

$$h_d[n] = \frac{1}{2\pi}\int_{-\Omega_0}^{+\Omega_0} \underbrace{H_d[\Omega]}_{=1} \cdot e^{j\Omega n} \cdot d\Omega =$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\Omega n}}{jn}\right]_{-\Omega_0}^{+\Omega_0} = \frac{1}{2\pi} \cdot \frac{e^{j\Omega_0 n} - e^{-j\Omega_0 n}}{jn} =$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$= \frac{1}{\pi}\,\frac{\sin \Omega_0 n}{n}$$

$$h_d[0] = \frac{\Omega_0}{\pi} = \frac{0{,}227\,\pi}{\pi} = 0{,}227$$

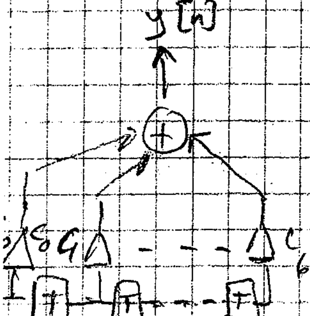
$$h_d[1] = \frac{\sin 0{,}227\pi}{\pi} = \leftarrow +0{,}208$$

$$h_d[-1] = \frac{\sin -0{,}227\pi}{\pi(-1)} = \leftarrow +0{,}208$$

$$h_d[+2] = h_d[-2] = \frac{\sin 0{,}227\pi \cdot 2}{\pi \cdot 2} = 0{,}1575$$

$$h_d[+3] = h_d[-3] = \frac{\sin 0{,}227\,\pi \cdot 3}{3\pi} = 0{,}089$$

Skifta 3 steg $\Rightarrow$ filterkoeff.



$h_{id}[0] = c_0 = \underline{0{,}089}$ $\quad h_{id}[1] = c_1 = \underline{0{,}158}$ $\quad h_{id}[2] = c_2 = \underline{0{,}208}$

$h_{id}[3] = c_3 = \underline{0{,}227}$ $\quad h_{id}[4] = c_4 = \underline{0{,}208}$ $\quad h_{id}[5] =$

$= c_5 = \underline{0{,}158}$ $\quad h_{id}[6] = c_6 = \underline{0{,}089}$

$$y[n] = \underbrace{h_{id}[0]}_{=c_0} \cdot x[n] + \underbrace{h_{id}[1]}_{=c_1} \cdot x[n-1] + \ldots + \underbrace{h_{id}[6]}_{=c_6}\, x[n-6]$$