Exercise 1

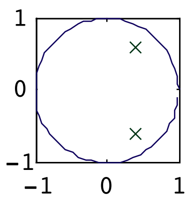
Difference equation; poles and zeros influence on the step and impulse respones; steady-state gain.

1. Systems of the type

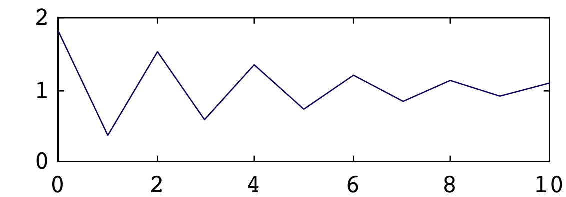
$$y(k) = K\frac{(1-z_1\text{q}^{-1})(1-z_2\text{q}^{-1})\ldots(1-z_m\text{q}^{-1})}{(1-\lambda_1\text{q}^{-1})(1-\lambda_2\text{q}^{-1})\ldots(1-\lambda_n\text{q}^{-1})}u(k)$$

are represented in A-F below where z_i, $i=1,\ldots,m$ are marked 'o' and λ_i, $i=1,\ldots,n$ are marked 'x'. Step responses (i.e. $y(k)$ when $u(k)=1$, $k\geq 0$) to four of the systems are marked 1 to 4. Combine the systems $A-F$ with the corresponding $1-4$ or alternative 5. *Hint:* First, derive the difference equations. Then, calculate the responses recursively a few instants.

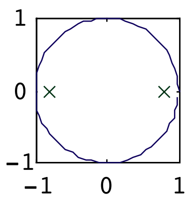
A



1.



B



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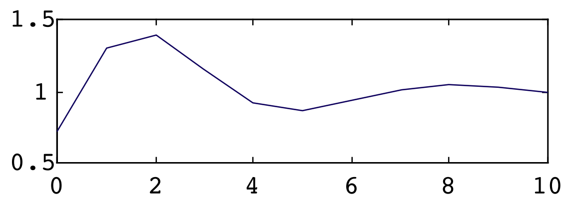

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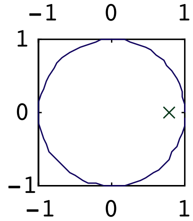
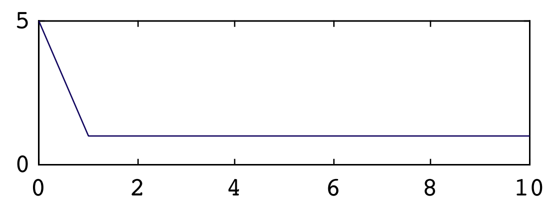


3.



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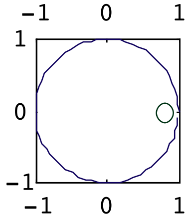
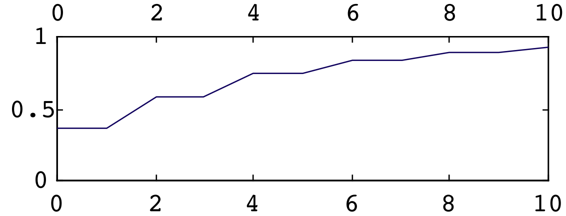
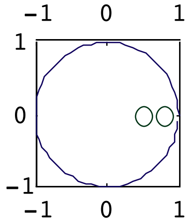


4.

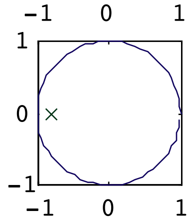


E



5. No match

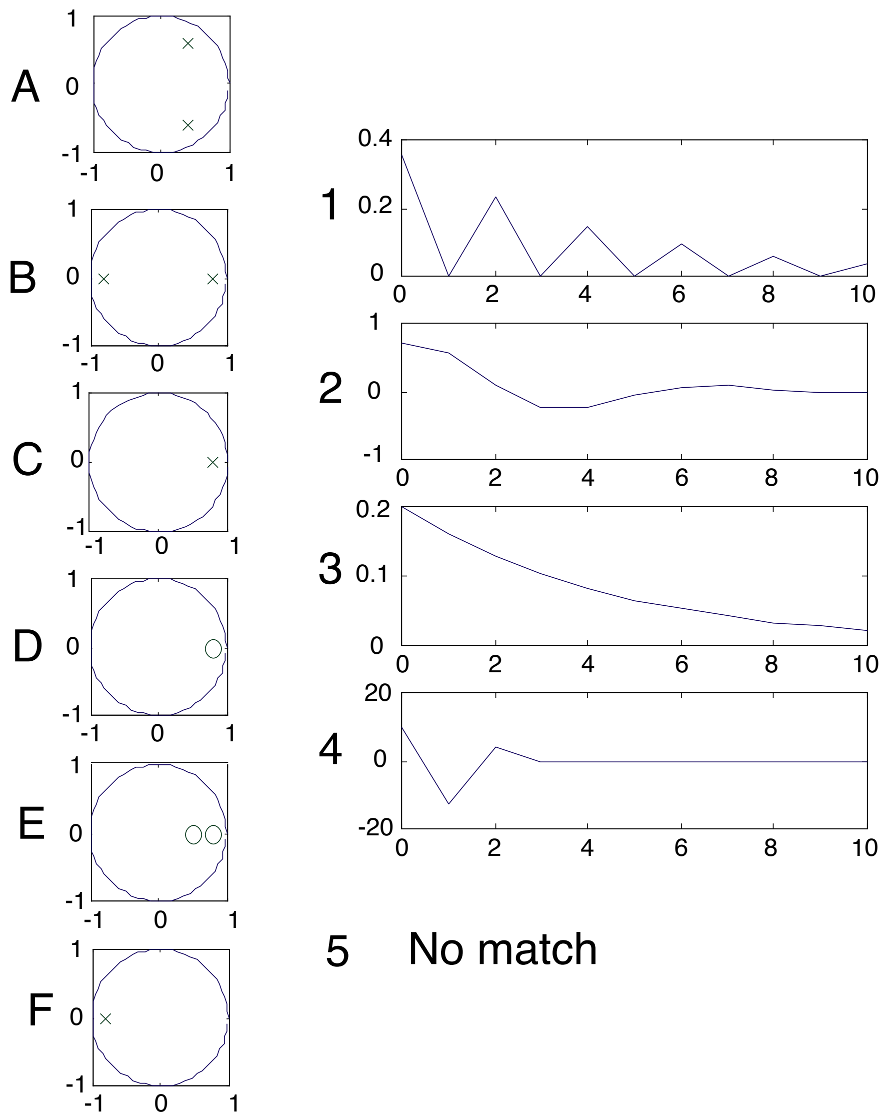
F



2. Systems of the type

$$y(k) = K\frac{(1 - z_1\mathrm{q}^{-1})(1 - z_2\mathrm{q}^{-1})\dots(1 - z_m\mathrm{q}^{-1})}{(1 - \lambda_1\mathrm{q}^{-1})(1 - \lambda_2\mathrm{q}^{-1})\dots(1 - \lambda_n\mathrm{q}^{-1})}u(k)$$

are represented in A-F below where z_i, $i = 1, \dots, m$ are marked 'o' and λ_i, $i = 1, \dots, n$ are marked 'x'. Pulse responses (i.e. $y(k)$ when $u(k) = 1$, $k = 0$ and $u(k) = 0$, $k \neq 0$) to four of the systems are marked 1 to 4. Combine the systems $A - F$ with the corresponding $1 - 4$ or alternative 5.



3. What are the steady-state (stationary) gain of the following systems?

a)

$$y(k) = \frac{0.1\mathrm{q}^{-1} - 0.2\mathrm{q}^{-2}}{1 - 0.1\mathrm{q}^{-1}} u(k)$$

b)

$$y(k) = 0.9y(k-1) + 0.1u(k-1)$$

c)

$$y(k) = \frac{-10(1 - 0.1\mathrm{q}^{-1})(1 - 0.2\mathrm{q}^{-1})}{(1 - 0.9\mathrm{q}^{-1})(1 - 0.8\mathrm{q}^{-1})} u(k)$$

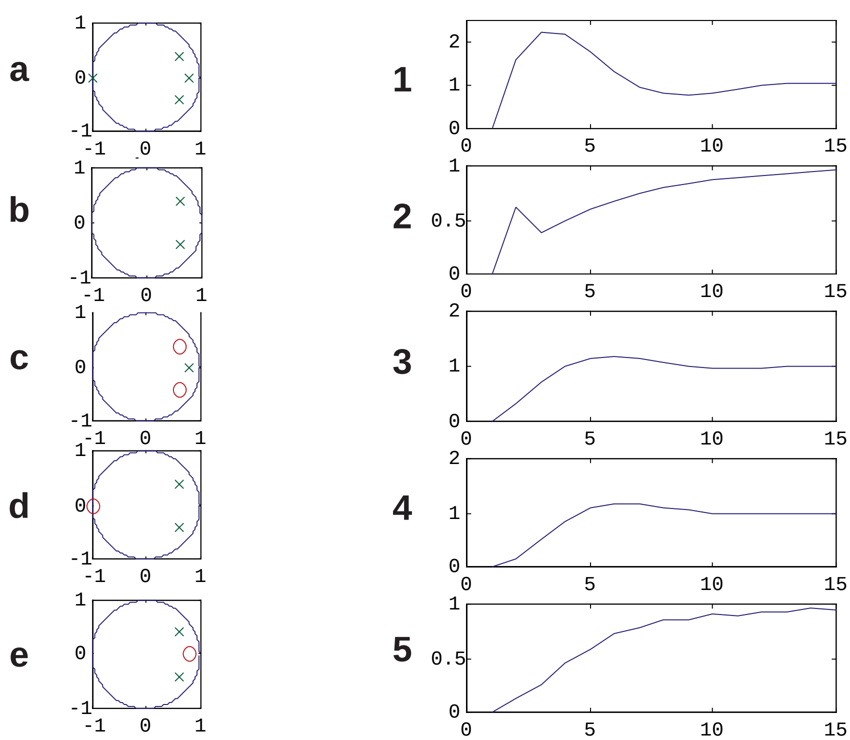
d)

$$y(k) - 2y(k-1) = 2u(k-10)$$

4. A process is described by

$$G(\mathrm{q}^{-1}) = K \frac{(1 - z_1\mathrm{q}^{-1}) \ldots (1 - z_m\mathrm{q}^{-1})}{(1 - \lambda_1\mathrm{q}^{-1}) \ldots (1 - \lambda_n\mathrm{q}^{-1})} \mathrm{q}^{-2}$$

In the Figures **a-e** below, z_k, $k = 1, \ldots, m$ and λ_k, $k = 1, \ldots, n$ are marked with 'o' and 'x', respectively, and unit step responses in Figures $1 - 5$. Combine the matching pairs between **a-e** and $1 - 5$.



Group problems

Each group should study a system on their own. All systems should satisfy the following

- one sample delay
- one zero at -1
- steady-state gain is 2
- two complex conjugated poles λ_1 and $\lambda_2 = conj(\lambda_1)$.

The location of the poles λ_1 are

Group	1	2	3	4	5	6	7
λ_1	0.9+0.2i	0.85+0.25i	0.8+0.3i	0.75+0.35i	0.7+0.4i	0.65+0.5i	0.6+0.55i

1. Illustrate the poles an zero in a pole-zero diagram (useful Sysquake functions: `scale equal; hgrid; plotroots`).
2. Describe the system as a recursive difference equation and show how the step response can be calculated recursively ten steps, $y(k)$, $k = 1, \ldots, 10$.
3. Use the backward-shift operator to describe the system in polynomial form. Then use this representation to calculate the step response using the Sysquake function: `filter`. Verify that it is the same as above.
4. Implement the difference equation as a Sysquake function by creating a sq-file (see seminar notes!). Verify that the step response becomes the same as in previous calculations.