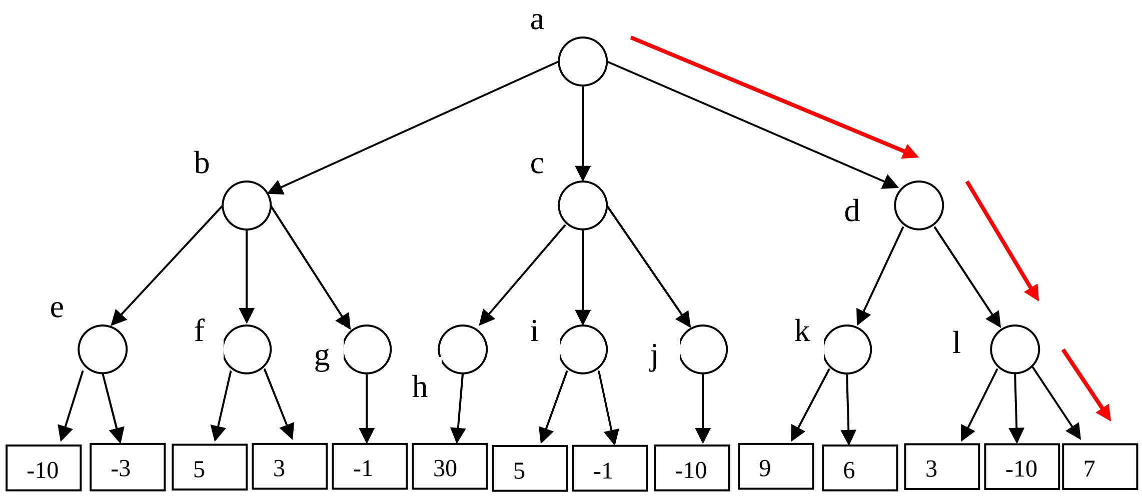
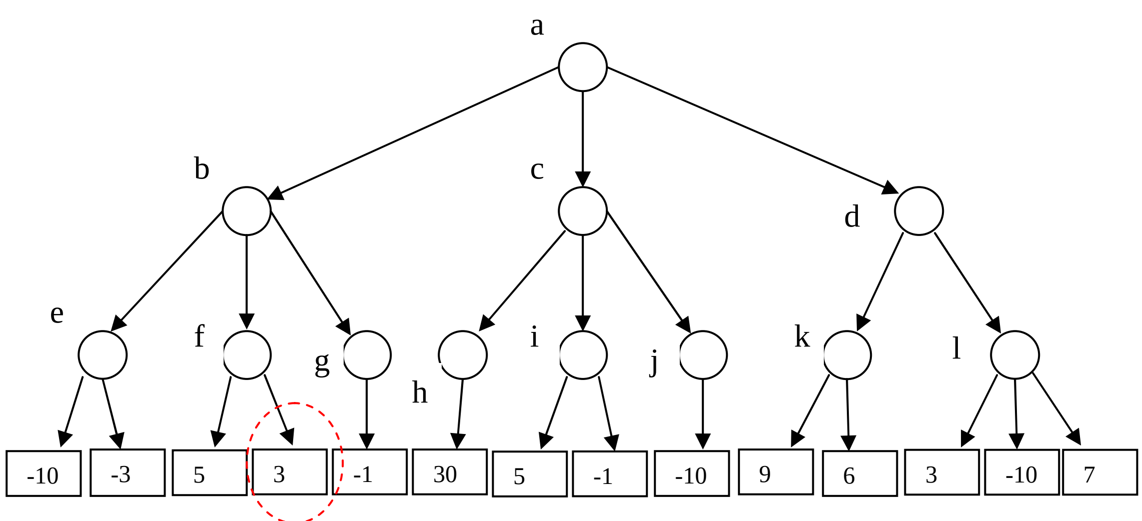
1 Game playing

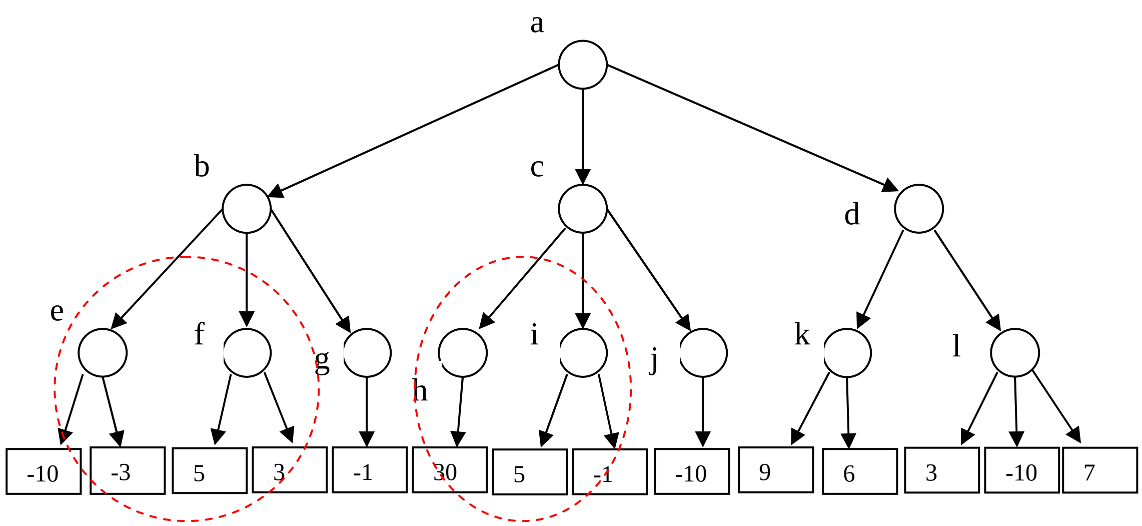
a.



b.



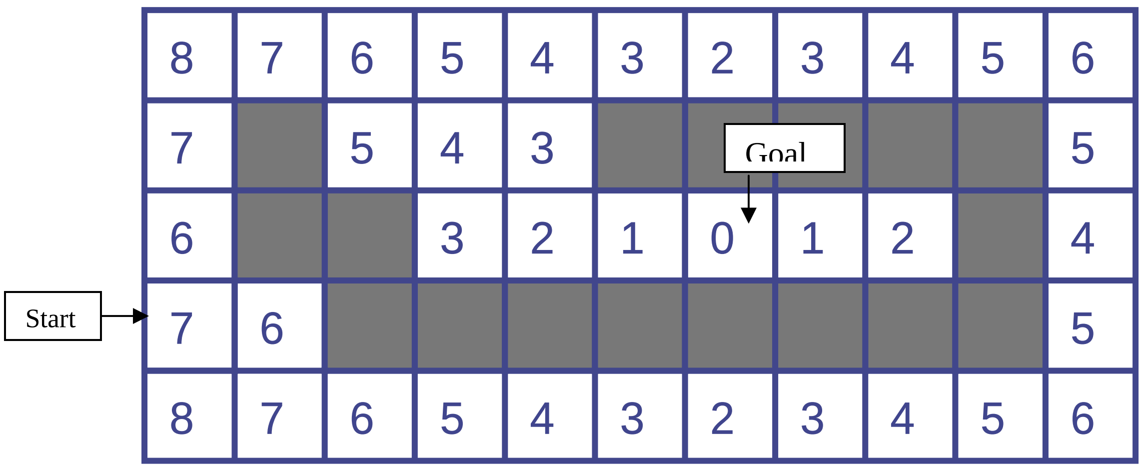
c.

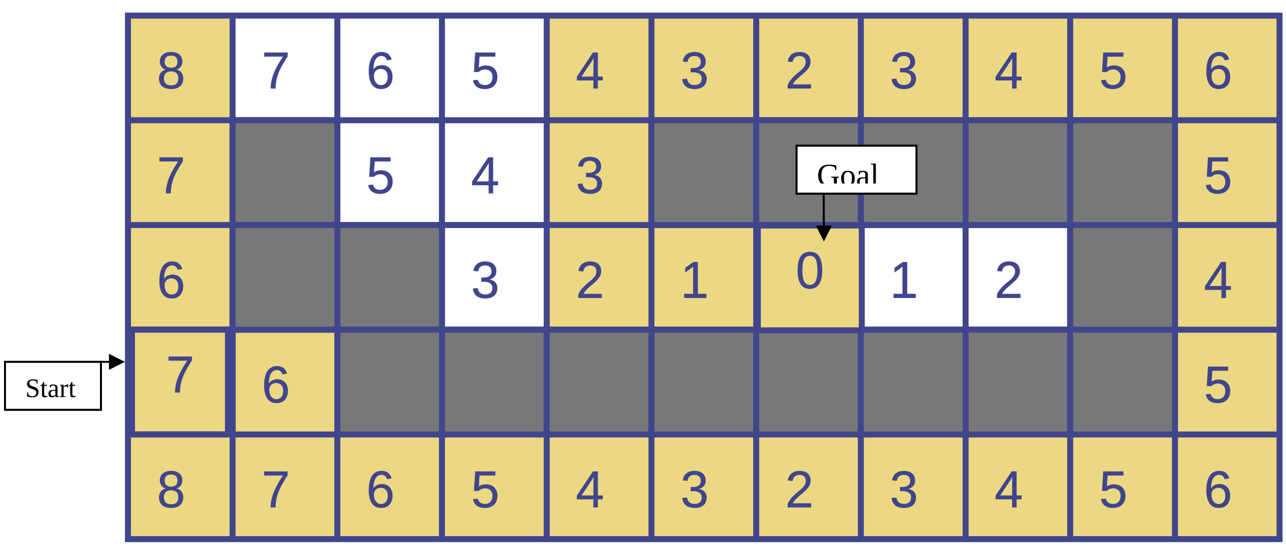


2 Informed search - Robot navigation

a. Greedy-best first search

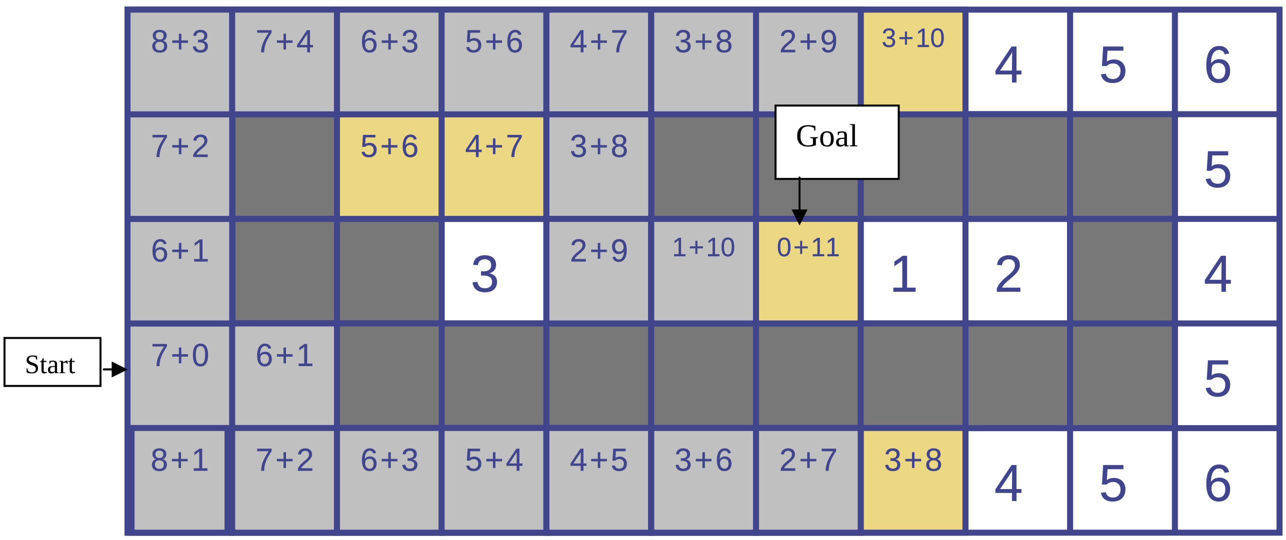
f(N) = h(N), with h(N) = Manhattan distance to the goal





b.

f(N) = h(N)+g(N), with h(N) = Manhattan distance to goal, g(N)=cost to reach state



c. Since greedy best-first search does not take into account the cost of reaching a certain state it ends up following a longer path.

d. Let h*(N) be the <u>true</u> cost of the optimal path from N to a goal node. Heuristic h(N) is admissible if: $0 \leq h(N) \leq h^*(N)$. An admissible heuristic is always optimistic

3 Logic

Represent the following sentences in first-order logic

a. A salesman has some apples, oranges and bananas.
b. All salesmen have some apples, oranges and bananas.
c. At least one salesman has some apples and oranges, but no bananas.
d. None of the salesmen have some apples, oranges or bananas.
e. For each fruit, there is a salesman that has at least one.

Solution: (one possible, others are also possible)

a. $Salesman(x) \Rightarrow \exists y\ Has(x, Apple(y)) \wedge \exists z\ Has(x, Orange(z)) \wedge \exists w\ Has(x, Banana(w))$

b. $\forall x\ \left[Salesman(x) \Rightarrow \exists y\ Has(x, Apple(y)) \wedge \exists z\ Has(x, Orange(z)) \wedge \exists w\ Has(x, Banana(w))\right]$

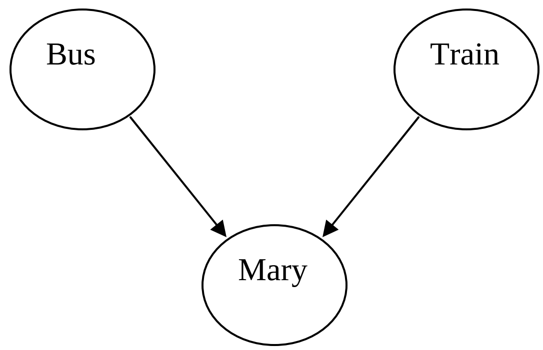
c. $\exists x\ \left[Salesman(x) \wedge \exists y\ Has(x, Apple(y)) \wedge \exists x\ Has(x, Orange(z)) \wedge \neg\forall w\ Has(x, Banana(w))\right]$

d. $\forall x\ \left[Salesman(x) \Rightarrow \neg\exists y\ Has(x, Apple(y)) \wedge \neg\exists z\ Has(z, Orange(z)) \wedge \neg\exists w\ Has(w, Banana(w))\right]$

e. $\forall x\ \left[Fruit(x) \Rightarrow \exists y\ Has(Salesman(y), x)\right]$

4. Bayesian network

a.



To introduce some notation...

P(bus=late) = P(¬B) = 0.3
P(bus=on time) = P(B) =0.7

P(train=late) = P(¬T) = 0.1
P(train=on time) = P(T) = 0.9

P(Mary=late | bus=on time, train=on time) = P(¬M | B, T) = 0.01
P(Mary=on time | bus=on time, train=on time) = P(M | B, T) = 1 – 0.01 = 0.99
P(Mary=late | bus=on time, train=late) = P(¬M | B, ¬T) = 0.9
P(Mary=on time | bus=on time, train=late) = P(M | B, ¬T) = 1 – 0.9 = 0.1
P(Mary=late | bus=late, train=on time) = P(¬M | ¬B, T) = 0.2
P(Mary=on time | bus=late, train=on time) = P(M | ¬B, T) = 1 – 0.2 = 0.8
P(Mary=late | bus=late, train=late) = P(¬M | ¬B, ¬T) = 0.9
P(Mary=on time | bus=late, train=late) = P(M | ¬B, ¬T) = 1 – 0.9 = 0.1

b.

From Bayes rule:

$$P(bus = late \mid Mary = late) = P(\neg B \mid \neg M) = \frac{P(\neg M \mid \neg B) \cdot P(\neg B)}{P(\neg M)} \qquad (1)$$

Marginalization of the numerator:

$$P(\neg M \mid \neg B) \cdot P(\neg B) = P(\neg M, \neg B) = P(\neg M, \neg B, \neg T) + P(\neg M, \neg B, T) \qquad (2)$$

From the Bayesian network:

$$P(\neg M \mid \neg B) P(\neg B) = P(\neg M, \neg B, \neg T) + P(\neg M, \neg B, T) =$$
$$P(\neg M \mid \neg B, \neg T) P(\neg B) P(\neg T) + P(\neg M \mid \neg B, T) P(\neg B) P(T) =$$
$$0.9 \cdot 0.3 \cdot 0.1 + 0.2 \cdot 0.3 \cdot 0.9 = 0.9 \cdot (0.03 + 0.06) \qquad (3)$$

Marginalization of the denominator:

$$P(\neg M) = P(\neg M, \neg B, \neg T) + P(\neg M, \neg B, T) + P(\neg M, B, \neg T) + P(\neg M, B, T) \qquad (4)$$

From the Bayesian network:

$$P(\neg M) = P(\neg M, \neg B, \neg T) + P(\neg M, \neg B, T) + P(\neg M, B, \neg T) + P(\neg M, B, T) =$$
$$P(\neg M \mid \neg B, \neg T) P(\neg B) P(\neg T) + P(\neg M \mid \neg B, T) P(\neg B) P(T) +$$
$$P(\neg M \mid B, \neg T) P(B) P(\neg T) + P(\neg M \mid B, T) P(B) P(T) =$$
$$0.9 \cdot 0.3 \cdot 0.1 + 0.2 \cdot 0.3 \cdot 0.9 + 0.9 \cdot 0.7 \cdot 0.1 + 0.01 \cdot 0.7 \cdot 0.9 =$$
$$0.9 \cdot (0.03 + 0.06 + 0.07 + 0.007) \qquad (5)$$

Combining (3) and (5) yields

$$P(bus = late \mid Mary = late) = \frac{P(\neg M \mid \neg B) \cdot P(\neg B)}{P(\neg M)} =$$
$$\frac{0.03 + 0.06}{0.03 + 0.06 + 0.07 + 0.007} = \frac{1}{1 + 0.077/0.09} = 54\% \qquad (6)$$