Laboration PM
Random processes STP855
University of Halmstad
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Simulating random phase and amplitude

We will simulate realisations of a process, $\{X_t\}$, where

$$X_t = A_0 + \sum_{k=1}^{9} A_k \cos(2\pi f_k t + \phi_k) \tag{1}$$

and $A_0 \in N(0, \sqrt{2})$ (thus $V(A_0) = 2$)
$A_k \in \mathit{Rayleigh}(\frac{1}{k}) \perp \phi_k \in R(0, 2\pi) \quad k = 1, 2, \ldots, 9$ (thus $V(A_k) = 1/k$)
$\{f_k\} = \{-0.5, -0.4, \ldots, 0.3\} \quad k = 1, 2, \ldots, 9$

For implementation, use the statistics software R.

Theoretical part

Assume that $A_k,\ k = 1, 2, \ldots, 9$ are variables that are Rayleigh distributed with parameter $\sigma_k^2 = 1/k$.

1. Show that $P(A_k \le a) = 1 - e^{-a^2 k/2}$ då $a > 0$ och $k = 1, 2, \ldots, 9$.

2. Let $F(a) = P(A_k \le a)$. Show that the inverse
   $$F^{-1}(u) = \sqrt{-\frac{2}{k}\log(1-u)} \quad \text{då } u \in (0, 1).$$

3. Assume $\mathcal{U} \in R(0, 1)$. Show that $F^{-1}(\mathcal{U}) \in \mathit{Rayleigh}(1/k)$.

4. Describe how to simulate the process $\{X_t\}$ (definded in (1)) as a Gaussian process.

Simulation part

To simulate this "random phase and aplitude" process one may use the results from the theory part. Below follows a suggested working procedure. The goal is to achieve (for instance 10) simulated samples of the process $\{X_t\}$ for the time-points $t = 0.01, 0.02, \ldots, 20.00$.

- Start by simulating a vector of 100 random numbers: $\boldsymbol{u} = (u(1), u(2), \ldots, u(100))$ by using the R-function `runif`.

- Then create 10 vectors (or a matrix with 10 rows)

$$\begin{array}{lcl} \boldsymbol{a}_0 & = & (a_{00}, a_{01}, \ldots, a_{09}) \\ \boldsymbol{a}_1 & = & (a_{10}, a_{11}, \ldots, a_{19}) \\ \vdots & & \\ \boldsymbol{a}_9 & = & (a_{90}, a_{91}, \ldots, a_{99}) \end{array}$$

  where $a_{j0},\ j = 0, 1, \ldots, 9$ are simulated observations of $A_0 \in N(0, 2)$

  and $a_{jk} = \sqrt{-\frac{2}{k}\log(1 - u(10j + k))}\,,\ \ j = 0, 1, \ldots, 9$ and $k = 1, 2, \ldots, 9$.

- Then simulate the vectors (or the matrix)

$$\begin{array}{lcl} \boldsymbol{\phi}_0 & = & (\phi_{00}, \phi_{01}, \ldots, \phi_{09}) \\ \boldsymbol{\phi}_1 & = & (\phi_{10}, \phi_{11}, \ldots, \phi_{19}) \\ \vdots & & \\ \boldsymbol{\phi}_9 & = & (\phi_{90}, \phi_{91}, \ldots, \phi_{99}) \end{array} \qquad \text{where } \phi_{jk} \in R(0, 2\pi).$$

- Finally, create the time vector $t = (0.01, 0.02, \ldots, 20.00)$ and the 10 simulated sequences where simulation number j ($j = 0, 1, \ldots, 9$) is

$$X_{j,t} = a_{j0} + \sum_{k=1}^{9} a_{jk}\cos(2\pi f_k t + \phi_{jk}).$$

Plot the 10 simulations, preferably in the same plot window (see `par(mfrow = ...  )`). Export the plot as postscript files (see `postscript(...)`).

Report writing

The laboration report should contain:

- Proofs of the theoretical results (clearly and concisely displayed)
- Program code together with comments and explenations on what have been done
- Printout of the plot

The laboration report should be returned to the examiner no later than ... 2004.

GOOD LUCK!