Introduction to the course Random Processes

The sequence $\{X_t : t \in T\}$ is called a **random process** (r.p.) with **parameter space** T if all random variables X_t are defined on the same sample space.
(A r.p. is usually denoted by $\{X_t\}$.)

Let $Z_1, Z_2, Z_3, \ldots$ be independent and $Z_k \in Po(\lambda)$ for all $k \in \mathbb{N}$.
Construct the process $\{X_t\}$ by letting $T = \mathbb{N}$ and $X_t = \sum_{k=1}^{t} Z_k$ for all $k \in \mathbb{N}$.
Then $\{X_t\}$ is called a **Poissonprocess** (Po-pr.).

Obs! Characteristics for a Po-pr. are

1) independent increments:
   $X_t - X_{t-1} = \sum_{k \leq t} Z_k - \sum_{k \leq t-1} Z_k = Z_t$
   $\Rightarrow\ X_2 - X_1,\ X_3 - X_2,\ X_4 - X_3,\ \ldots$ ober.
   $\Rightarrow\ X_{t_2} - X_{t_1} \perp X_{s_2} - X_{s_1}$ for all disjoint intervals $(t_1, t_2)\,,\, (s_1, s_2) \subset \mathbb{N}$.
2) stationary increments
   $X_2 - X_1 \overset{\mathcal{D}}{=} X_3 - X_2 \overset{\mathcal{D}}{=} \ \ldots\ \overset{\mathcal{D}}{=} Z_k \in Po(\lambda)$
   $\Rightarrow\ X_{t+h} - X_t = \sum_{k=t+1}^{t+h} Z_k \in Po(h\lambda)$
   (since X, Y independent, $Po(\lambda) \Rightarrow\ X + Y \in Po(2\lambda)$)
   i.e. only a function of the time distance h

The sequence $\{X_t\}$ is a 1-dim. **Gaussian process**
if all linear combinations $a_1 X_{t_1} + \ldots + a_n X_{t_n}$
are normally distributed for all n and $t_1, \ldots, t_n \in T$.

For example (white) noise: $T = \mathbb{N},\ X_t \in N(m, \sigma),\ X_i \perp X_j$.