Additional exercises

1. Calculate $m(t)$, $R(s,t)$ of the process $\{X_t\}$ and determine wether it is stationary (strongly, weakly or not at all) when

   a) $X_t = At + B$
   where $A \perp B$ and $E(A) = a \quad V(A) = \sigma_A^2$
   $E(B) = b \quad V(B) = \sigma_B^2$

   b) $X_t = At^2 + Bt + C$
   where $A \perp B \perp C$ and $E(A) = E(B) = E(C) = 1$
   $V(A) = V(B) = V(C) = 1$

   c) $X_t = \cos(at + B)$
   where $a \in \mathbb{R},\ B \in U(0, 2\pi)$

   d) $X_t = \frac{1}{n} \sum_{k=1}^{n} g(t, A_k),\ 0 \le t \le 1$
   where $A_1, \ldots, A_n$ indep. and $A_i \in U(0,1)$
   and $g(t,x) = \begin{cases} 1 & x \le t \\ 0 & x > t \end{cases}$

2. Assume that $\{B_t\}$ is a Brownian motion. Calculate $m(t)$, $R(s,t)$ of the process $\{X_t\}$ and and determine wether it is stationary (strongly, weakly or not at all) when

   a) $X_t = tB_{1/t},\ t > 0$

   b) $X_t = a^{-1} B_{a^2 t},\ t \ge 0$

   c) $X_t = (B_t)^2,\ t \ge 0$

   d) $X_t = B_t - tB_1,\ 0 \le t \le 1$

3. Let $X_t = \sin(At),\ A \in U(0, 2\pi)$

   a) Show that $\{X_t : t \in \mathbb{N}\}$ is weakly but not strictly stationary.

   b) Is $\{X_t : t \in [0, \infty)\}$ just weakly stationary or even strictly stationary?

4. Let $X_n = \sum_{k=1}^{N} \sigma_k \sqrt{2}(a_k n - U_k)$ where
   σ_k and a_k are positive real numbers for all $k = 1, \ldots, N$
   $U_1, \ldots, U_N$ are independent and $U_k \in U(0, 2\pi)$ for all $k = 1, \ldots, N$.
   Show that $\{X_t\}$ is weakly stationary.