Repetition of elementary mathematical statistics

Definitions

P is a **probability measure** if
1. $0 \le P(A) \le 1$
2. $P(\Omega) = 1$
3. $A \cap B = \emptyset \;\Rightarrow\; P(A \cup B) = P(A) + P(B)$

for all events $A, B \subset \Omega$ where Ω is the entire sample space.
(**obs!** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$)

Conditional probability $P(A|B) = P(A \cap B)/P(B)$

A, B **independent** if $P(A \cap B) = P(A)P(B)$
(**obs!** if $P(A|B) = P(B)$ and vice versa)

DISCRETE	**probability function**	$p(x) = P(X = x)$ $P(X \le a) = F(a) = \sum_{x \le a} p(x)$
	expected value	$E(X) = \sum_{x\in\Omega} xp(x) =: \mu$ $E(g(X)) = \sum_{x\in\Omega} g(x)p(x)$ $E(XY) = \sum_{x\in\Omega_x} \sum_{x\in\Omega_y} xy\, p(x,y)$
	variance	$V(X) = \sum_{x\in\Omega}(x-\mu)^2 p(x) =: \sigma^2$
CONT.	**density function**	$f(x) = \frac{d}{dx} P(X \le x)$ $P(X \le a) = F(a) = \int_{-\infty}^{a} f(x)\, dx$
	expected value	$E(X) = \int_\Omega x f(x)\, dx =: \mu$ $E(g(X)) = \int_\Omega g(x) f(x)\, dx$ $E(XY) = \int_{\Omega_x} \int_{\Omega_y} xy f(x,y)\, dx\, dy$
	variance	$V(X) = \int_\Omega (x-\mu)^2 p(x) =: \sigma^2$
BOTH	**covariance**	$C(X,Y) = E((X-\mu_x)(Y-\mu_y))$
	correlation	$\rho(X,Y) = C(X,Y)/\sqrt{V(X)V(Y)}$

Calculation rules

$E(aX + bY) = aE(X) + bE(Y)$ for all random variables X, Y and constants a, b.

If X, Y independent, then $V(aX + bY) = a^2V(X) + b^2V(Y)$.

$V(X) = E(X^2) - E(X)^2$

$C(X,Y) = E(XY) - E(X)E(Y)$