Random processes STP855
University of Halmstad
Examiner: Eric Järpe

Introduction exercises

In the following exercises, assume that the index set is $T = \mathbb{N}$.

1. Let $X = \{X_t\}$ be a Poisson process with parameter (λ). Calculate
   (a) $E(X_t)$
   (b) $V(X_t)$
   (c) $C(X_t, X_{t+1})$
   (d) $C(X_t, X_s)$
   (e) $P(X_t \leq \lambda t)$ when $\lambda = 0.005$ and $t = 1000$

   Let $Y_t = \frac{1}{\sqrt{\lambda t}}(X_t - \lambda t)$. Calculate
   (f) $E(Y_t)$
   (g) $C(Y_t, Y_s)$
   (h) approximately $P(Y_t > a)$ when $\lambda = 0.1,\ t = 160,\ a = -1$

2. Let $X_t = \sin^2(\frac{\pi t}{32}) + \cos^2(\frac{\pi t}{32})Z_t$ where $\{Z_t\}$ are independent and distributed $N(1, 1)$. Calculate
   (a) $E(X_t)$
   (b) $C(X_t, X_s)$
   (c) the smallest number t such that $4V(X_t) = 1$

3. Let $X_0 \in N(0, 1)$ and recursively $X_t = bX_{t-1} + Z_t$ for $t \geq 1$ where $\{Z_t\}$ are independent and distributed $N(0, \sigma_Z)$. Calculate
   (a) $C(X_t, X_{t+h})$
   (b) σ_Z such that $V(X_t) = 1$
   (c) the conditional probability that $X_t \leq 4$ given $X_{t-1} = 4.5$ when $b = 0.9$ and $V(X_t) = 1$
   (d) the joint density function for $X_0, X_1, \ldots, X_t$: $f(x_0, x_1, \ldots, x_t)$ if $\sigma_Z = 1$ and $b = -0.5$