Theorem 1

(*a*) $G = \mathcal{F}(g)$ *and* $h(\tau) \equiv G(\tau) \;\Rightarrow\; H(f) \equiv g(-f)$

(*b*) $g = \mathcal{F}^{-1}(G)$ *and* $H(f) \equiv g(f) \Rightarrow\; h(\tau) \equiv G(-\tau)$

Proof:

(*a*) Assume $G = \mathcal{F}(g)$.
Then $g(-\tau) = \mathcal{F}^{-1}(G)(-\tau) = \int_{\mathbb{R}} e^{i2\pi f(-\tau)} G(f)\,df = \int_{\mathbb{R}} e^{-i2\pi f\tau} G(f)\,df$,
i.e. $g(-f) = \int_{\mathbb{R}} e^{-i2\pi f\tau} G(\tau)\,d\tau = \mathcal{F}(G)(f)$.
Now, if $h(\tau) \equiv G(\tau)$, then $H(f) = \mathcal{F}(h)(f) = \mathcal{F}(G)(f) = g(-f)$.

(*b*) In analogous fashion we have that $G(-f) = \mathcal{F}(g)(-f) = \int_{\mathbb{R}} e^{i2\pi f\tau} g(\tau)\,d\tau$,
so $G(-\tau) = \mathcal{F}^{-1}(g)(\tau)$. Now, if $H(f) \equiv g(f)$,
then $h(\tau) = \mathcal{F}^{-1}(H)(\tau) = \mathcal{F}^{-1}(g)(\tau) = G(-\tau)$.

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