Exam for Random Processes, 5p

January 18, 2003, 9.00 am − 13.00 pm

Max number of points: 30. **Bounds**: 12p ⇒ grade 3, 18p ⇒ grade 4, 24p ⇒ grade 5.
Allowed aids:
Sheet of formulae attached to the exam, calculator and Mathematics Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53).

For each problem a *complete* solution should be given.
All solutions should be thoroughly presented.
Each soloution should start at the top of a new sheet of paper.
Only one solution a sheet.
The proper solutions will be available on internet at
`http://www.hh.se/staff/erja` → `teaching` → `random processes` → `030118: solution`

1. Show that <u>if</u> $G = \mathcal{F}(g)$ and $h(\tau) \equiv G(\tau)$ <u>then</u> är $H(f) \equiv g(-f)$ (where $H = \mathcal{F}(h)$ and $g = \mathcal{F}^{-1}(G)$). (3p)

2. Show that <u>if</u> $\{X_t : t \in \mathbb{R}\}$ is weakly stationary and $Z_t = X_t$ för $t \in \{dk : k \in \mathbb{Z}\}$ <u>then</u> $R_Z(f) = \sum_{-\infty}^{\infty} R_X(f + \frac{k}{d})$ for $-\frac{1}{2d} < f < \frac{1}{2d}$. (4p)

3. Assume that $\{X_t\}$ och $\{Y_t\}$ are mutually independent Poisson processes with parameter $\lambda_X = 0.1$ and $\lambda_Y = 0.2$ respectively.
   (a) Calculate $P(X_{200} < Y_{101})$ approximatively. (3p)
   (b) Show that the process $Z_t = X_{2t} - Y_t$ is not stationary. (3p)

4. The spectral density of the process $\{X_t\}$ is $R_X(f) = |f|e^{-af^2}$. This signal will be sampled from with interval 0.3. What does a have to be for not risking that more than 10% of the signal is distorted? (3p)

5. Assume that $\{X_t\}$ is an $AR(1)$ process with regression parameter $a_1 = -2/3$ and noise process $\{\epsilon_t\}$ where $\epsilon_t \in N(0, 1/2)$.
   (a) What does $\mathsf{E}(X_t)$ and $\mathsf{V}(X_t)$ have to be for the process $\{X_t\}$ to be strictly stationary? (3p)
   (b) Calculate expectation function and covariance function of the derivative process $\{X'_t\}$. (3p)

6. Let the expectation function of $\{X_t\}$ be $m_X = 0$ and the covariance function be $R_X(\tau) = \max(0, \frac{1}{2} - |\tau|)$.
   (a) Calculate the spectral density $R_X(f)$. (By all means, use the table!) (3p)
   (b) The signal $\{Y_t\}$ is achieved by a linear transfer defined by $Y_t = \int_{t-1}^{t} X_u \, du$. Derive the spectral density of $\{Y_t\}$. (5p)

GOOD LUCK!