Exam for Random Processes, 5p

August 13, 2003, 9.00 am − 13.00 pm

Max number of points: 30. **Bounds**: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
Allowed aids: Sheet of formulae attached to the exam, calculator and Mathematics Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53).

For each problem a *complete* solution should be given.
All solutions should be thoroughly presented.
Each soloution should start at the top of a new sheet of paper.
Only one solution a sheet.
The proper solutions will be available on internet at
`http://www.hh.se/staff/erja` →`teaching` →`random processes` →`030813: solution`

1. Show that <u>if</u> $\boldsymbol{X}$ is n-dimensionally normally distributed with uncorrelated vector elements, <u>then</u> the vector elements are independent. (3p)

2. Show that <u>if</u> $\{X_t\}$ is weakly stationary, H is a transfer function[1] for a linear filter, $\{Y_t\}$ is the filtered process and R_X and R_Y are the respective spectral densities[2], <u>then</u> $R_Y(f) = |H(f)|^2 R_X(f)$. (4p)

3. Let $\{X_t\}$ be a Poisson process with intensity $\lambda = 0.02$ and calculate
   (a) $P(X_t - X_{t-25} > 1)$ for $t \geq 26$. (2p)
   (b) $P(X_{1000}^2 \leq 100)$ approximately. (2p)
   (c) Let $Y_t = (X_t - 0.02t)/\sqrt{0.02t}$ for $t \geq 1$. Is $\{Y_t\}$ a stationary process? If so, show that. If not, give a counterexample. (3p)

4. Let $\{X_t\}$ be an $MA(2)$-process defined by $X_t = \epsilon_t + 2\epsilon_{t-1} - 0.5\epsilon_{t-2}$ where $\{\epsilon_t\}$ is white noise with $\sigma_\epsilon^2 = 1$. Calculate
   (a) $P(X_t - X_{t-1} > 1)$. (3p)
   (b) the covariance function $r_X(\tau)$. (3p)

5. A signal has spectral density $|f|e^{-c|f|}$. The signal is to be sampled from with sampling interval 0.5. Determine approximately the value of c such that the error margin of the sampled signal is less than 5%. (4p)

6. Let $\{X_t\}$ be a process with mean $m_X = 0$ and spectral density $R_X(f) = \frac{1}{2}\delta_{-1}(f) + \frac{1}{2}\delta_1(f)$. Calculate the covariance function of
   (a) $\{X_t\}$. (3p)
   (b) the derivative process $\{X_t'\}$. (3p)

GOOD LUCK!

[1] På svenska: *frekvenfunktion*

[2] På svenska: *spektraltätheter*