Exam for Random Processes, 5p

October 25, 2002, 9.00 am − 13.00 pm

Max number of points: 30. **Bounds**: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
Allowed aids:
Sheet of formulae attached to the exam, calculator and Mathematics Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53).

For each problem a *complete* solution should be given.
All solutions should be thoroughly presented.
Each soloution should start at the top of a new sheet of paper.
Only one solution a sheet.
The proper solutions will be available on internet at
`http://www.hh.se/staff/erja` → `teaching` → `random processes` → `021025: solution`

1. Show that all weakly stationary stationary normal processes are strictly stationary. (4p)

2. Assume $\{X_t\}$ is a weakly stationary process differentiable in squared mean and that $\{X_t\}$ has a $R_X(\tau)$. Show that $m_{X'}(t) = 0$ and that $r_{X'}(\tau) = -r''_X(\tau)$. (4p)

3. Let $\{X_t\}$ be a Poisson process with intensity 0.1 and let $\{Y_t\}$ be defined by $Y_t = 4X_{t-1} - 3X_t - 7,\ t = 2, 3, 4, \ldots$
   (a) Calculate the covariance function of $\{Y_t\}$. (3p)
   (b) Calculate $P(3Y_{151} + 4Y_{150} > 16X_{149})$ approximately. (4p)

4. Let $\{X_t\}$ be an AR(2)-process defined by $X_t = aX_{t-1} - 0.5X_{t-2} + \epsilon_t$.
   (a) What is the condition on a for $\{X_t\}$ to be stationary? (4p)
   (b) Calculate a if $V(X_t) = 7$ and $C(X_t, X_{t+1}) = -5.6$. (2p)

5. A signal has spectral density $R(f) = 1/(1 + 4f^2)$. How large proportion of the signal is transferred correctly for certain if it is sampled with interval 0.1? (4p)

6. Let $\{X_t\}$ be a weakly stationary process with expectation function $m_X = \frac{1}{2}$ and covariance function $r_X(\tau) = \begin{cases} \frac{1}{2}|\tau| & \text{if } |\tau| \leq 2 \\ 0 & \text{otherwise.} \end{cases}$ Assume further that $\{X_t\}$ is transferred with impulse response $h(t) = \delta_{-1}(t) + \delta_1(t)$.
   (a) Is the transfer causal? (1p)
   (b) Calculate expectation function and covariance function of the transferred signal. (4p)

GOOD LUCK!