Tentamen i Stokastiska Processer, 5 poäng

January 16, 2004, 13.30 – 17.30

Max number of points: 30. **Bounds**: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
Allowed aids: Sheet of formulae attached to the exam, calculator and Mathematics Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53).

For each problem a *complete* solution should be given.
All solutions should be thoroughly presented.
Each soloution should start at the top of a new sheet of paper.
Only one solution a sheet.
The proper solutions will be available on internet at
`http://www.hh.se/staff/erja` → `teaching` → `random processes` → `040116: solution`

1. Show that a weakly stationary Gaussian process is strongly stationary. (3p)

2. Let the process $\{X_t\}$ be defined by $X_t = bX_{t-1} + \epsilon_t$ for all $t \in \mathbb{Z}$ where $\{\epsilon_t\}$ is an independent sequence where $\epsilon_t \in N(0, \sigma_\epsilon^2)$. Show that if $\{X_t\}$ is weakly stationary, then $|b| \leq 1$. (3p)

3. Let $\{X_t : t \in \mathbb{R}\}$ be shot noise with intensity 35 and impulse function
$$g(t) = \begin{cases} 2t^3 & \text{if } 0 \leq t \leq 1 \\ 0 & \text{o.w.} \end{cases}$$
Calculate
   (a) expectation and covariance function of $\{X_t\}$. (3p)
   (b) approximately the probability $P(X_{100} > 20)$. (Motivate any approximations!) (3p)

4. For which values of α is $r(\tau)$ a covariance function in discrete time if
$$r(\tau) = \begin{cases} \alpha & \text{if } \tau = 0 \\ \alpha^2 & \text{if } |\tau| = 3 \\ 0 & \text{o.w.} \end{cases}$$
(3p)

5. Let $\{X_t : t \in \mathbb{R}, t \geq 0\}$ be a Poisson process with intensity 1. Construct the process $\{Y_t\}$ by $Y_t = 0.5^t X_{4^t} - 2^t$ for all $t \in \mathbb{R}$.
   (a) Show that $\{Y_t\}$ is weakly stationary. (3p)
   (b) Calculate $P(2Y_1 = Y_0 + 1)$. (3p)

6. Let $\{X_t\}$ be a weakly stationary process with $r_X(\tau) = \frac{\sin(2\pi\tau)}{2\pi\tau}$ and construct $\{Y_t\}$ by $Y_t = \theta X_t + (1-\theta)X_{t-1}$ for all $t \in \mathbb{R}$ where $\theta \in (0, 1)$.
   (a) Determine the impulse response which filters $\{X_t\}$ into $\{Y_t\}$. (3p)
   (b) Derive the spectral density of the derivative process $\{Y_t'\}$. (3p)
   (c) Consider only the frequency $f = \frac{1}{2}$. For which value of θ is the spectral density of Y_t minimized (at $f = \frac{1}{2}$)? (3p)

GOOD LUCK!