# Exam for Random Processes, 5 points/7.5 ECTS

August 14, 2004, 9 am – 1 pm

**Max number of points**: 30. **Bounds**: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
**Allowed aids**: Sheet of formulae attached to the exam, calculator and Mathematics Handbook: Beta.
**Examiner**: Eric Järpe (035-16 76 53).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://www.hh.se/staff/erja` → `Teaching` → `Random processes` → `040814: Solution`

1. Show that if $\boldsymbol{X} = (X_1, X_2, \ldots, X_n)$ is $n$-dimensionally normally distributed with uncorrelated vector elements $X_i$, then $X_1, X_2, \ldots, X_n$ are all independent of each other. (3p)

2. Assume that $U$ is uniformly distributed on $[0, 1]$. Show that $X = \sqrt{-2\ln(1-U)}$ has density function $f(x) = x\,e^{-x^2/2}$ for all $x > 0$ (i.e. $X$ is *Rayleigh distributed*). (3p)

3. Let $\{X_t\}$ be an $AR(1)$ process defined by $X_t + 0.5X_{t-1} = \epsilon_t$ where $\{\epsilon_t\}$ is white noise and $V(X_t) = 2$. Determine the probability $P(X_t - 2X_{t-1} \leq 1)$. (3p)

4. Let $\{X_t : t \in \mathbb{R}\}$ be a process with expectation function $m_X(t) = 1$ and covariance function $r_X(\tau) = e^{-|\tau|}$.
   (a) Calculate the spectral density function of $\{X_t\}$. (3p)
   $\{X_t\}$ is sampled: $Z_t = X_t$ for $t \in \mathbb{Z}$.
   (b) Calculate approximately the spectral density of the sampled process $\{Z_t\}$. (3p)
   Now, let $\{Y_t\}$ be defined by $Y_t = X_t + X_{t-1}$ for all $t \in \mathbb{R}$.
   (c) Show that $\{Y_t\}$ is weakly stationary. (4p)
   In an experiment, 8 observations of $\{X_t\}$ are about to be made.
   (d) Calculate approximately the variance of the mean value estimator. (3p)
   Suppose the values 1.73 1.85 2.25 1.89 0.82 1.99 1.71 2.11 are observed.
   (e) Estimate the expected value function $m_X(t)$. (2p)

5. Let $\{X_t : t \in \mathbb{R}\}$ be shot noise with intensity $\lambda = 100$ and impulse function
   $$g(t) = \begin{cases} e^t & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$
   (a) Calculate the covariance function of $\{X_t\}$. (3p)
   (b) Determine the probability $P(X_t + X_{t+0.5} > 100)$. Motivate any approximations. (3p)

*GOOD LUCK!*