Exam for Random Processes, 5 points/7.5 ECTS

April 16, 2004, 9 am – 1 pm

Max number of points: 30. **Bounds**: 12p ⇒ grade 3, 18p ⇒ grade 4, 24p ⇒ grade 5.
Allowed aids: Sheet of formulae attached to the exam, calculator and Mathematics Handbook: Beta.
Examiner: Eric Järpe (035-16 76 53).

For each problem a *complete* solution should be given.
All solutions should be thoroughly presented.
Each soloution should start at the top of a new sheet of paper.
Only one solution a sheet.
The proper solutions will be available on internet at
`http://www.hh.se/staff/erja` → `teaching` → `random processes` → `040416: solution`

1. Show that if a weakly stationary process $\{X_t\}$ is differentiable in squared mean, then the covariance function of the derivative process $\{X_t'\}$ satisfies $r_{X'}(\tau) = -r''_X(\tau)$ for all τ. (3p)

2. Show that if $\{X_t : t \in \mathbb{R}\}$ is weakly stationary and $Z_t = X_t$ for all $t = 0, \pm 1, \pm 2, \ldots$, then the spectral density function of the process $\{Z_t\}$ is $R_Z(f) = \sum_{k\in\mathbb{Z}} R_X(f + \frac{k}{d})$ where $-\frac{1}{2d} < f \leq \frac{1}{2d}$. (4p)

3. Let $\{X_t\}$ be a random process with covariance function $r(\tau) = \sqrt{2\pi}e^{-2\pi^2\tau^2}$. How large error due to the alias effect is risked by sampling $\{X_t\}$ at time-points $t = 0, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm 1, \ldots$? (Hint: Use the table of standard normal percentiles!) (4p)

4. Let $\{X_t : t \in \mathbb{R}\}$ be a process with covariance function $r_X(\tau) = 1/(4 + \tau^2)$.
   (a) Derive the spectral density function of the derivative process $\{X_t'\}$. (3p)
   (b) Let $\{Y_t\}$ be the process defined by $Y_t = X_t - X_{t-1}$ for all t. Determine the cross-covariance function of $\{X_t\}$ and $\{Y_t\}$: $r_{X,Y}(t, t+\tau) = C(X_t, Y_{t+\tau})$. (3p)

5. Bob is sometimes at his office and sometimes somewhere else during his working hours. Let $\{X_t\}$ be a Poisson process with intensity 1. Then a process describing wether Bob is in or outside his office at time t is $\{Y_t\}$ where $Y_t = (-1)^{X_t}$. Determine the covariance function of $\{Y_t\}$. (4p)

6. Let $\{X_t\}$ be an $MA(2)$ process satisfying $X_t = a\epsilon_t - \epsilon_{t-2}$ for all $t \in \mathbb{Z}$ where $V(X_t) = 1$ and $\{\epsilon_t\}$ is normally distributed white noise with $V(\epsilon_t) = \sigma_\epsilon^2$.
   (a) What is the variance σ_ϵ^2? (3p)
   (b) What is the probability $P(X_t + X_{t+2} > 1)$ if $a = 3$? (3p)
   (c) Determine $r_X(\tau)$, the covariance function of $\{X_t\}$. (3p)

GOOD LUCK!