# Exam for Random Processes, 5 points/7.5 ECTS

January 4, 2005, 9 am – 1 pm

**Max number of points**: 30.
**Halmstad University grading bounds**: 12p ⇒ grade 3, 18p⇒ grade 4, 24p ⇒ grade 5.
**ECTS bounds**: 12p ⇒ grade E, 15p ⇒ grade D, 18p ⇒ grade C, 21p ⇒ grade B, 24p ⇒ grade A.
**Allowed aids**: Sheet of formulae attached to the exam, calculator and Mathematics Handbook: Beta.
**Examiner**: Eric Järpe (035-16 76 53).

For each problem a *complete* solution should be given. All solutions should be thoroughly presented. Each solution should start at the top of a new sheet of paper. Only one solution a sheet. The proper solutions will be available on the internet at
`http://www.hh.se/staff/erja` → `Teaching` → `Random processes` → `050104: Solution`

1. Assume that $\{X_t\}$ is a weakly stationary process with cvf $r_X$, and that $r_X = G$ where $G$ is the Fourier transform of $g$. Show that the spectral density function of $\{X_t\}$ is $R_X = g$. (3p)

2. Show that a weakly stationary Gaussian process is strongly stationary. (3p)

3. Suppose $\{X_t\}$ is a weakly stationary process with ef $m_X = 1$ and spectral density function $R_X(f) = \sqrt{\frac{1}{4}e^{-|f|}}$.
   (a) Determine the cvf $r_X(\tau)$. (2p)
   (b) Assuming that $\{X_t\}$ is a Gaussian process, calculate the probability $P(X_t + X_t' \leq 5)$. (4p)

4. Let $r_X(\tau) = \sum_{k\in\mathbb{Z}} \frac{\delta_k(\tau)}{2^{|\tau|}}$ be the cvf of the weakly stationary process $\{X_t : t \in \mathbb{R}\}$.
   (a) Calculate $R_X(0)$. (4p)
   (b) Suppose that $\{X_t\}$ is filtered with the response function $h(t) = \delta_c(t)$ (where $c \in \mathbb{R}$) resulting in the output process $\{Y_t\}$. Calculate the cvf $r_Y$. (4p)

5. Let $\{X_t\}$ be an $MA(q)$ process defined by $X_t = \sum_{k=0}^{q}(-1)^k\epsilon_{t-k}$ where $\sigma_\epsilon^2 = 1$.
   (a) Calculate the cvf $r_X$. (4p)
   (b) What kind of process $\{Y_t\}$ is achieved by by letting $Y_t = X_t + X_{t-1}$. (3p)
   (c) Calculate the spectral density function $R_Y$ in the case when $q = 99$. (3p)

*GOOD LUCK!*